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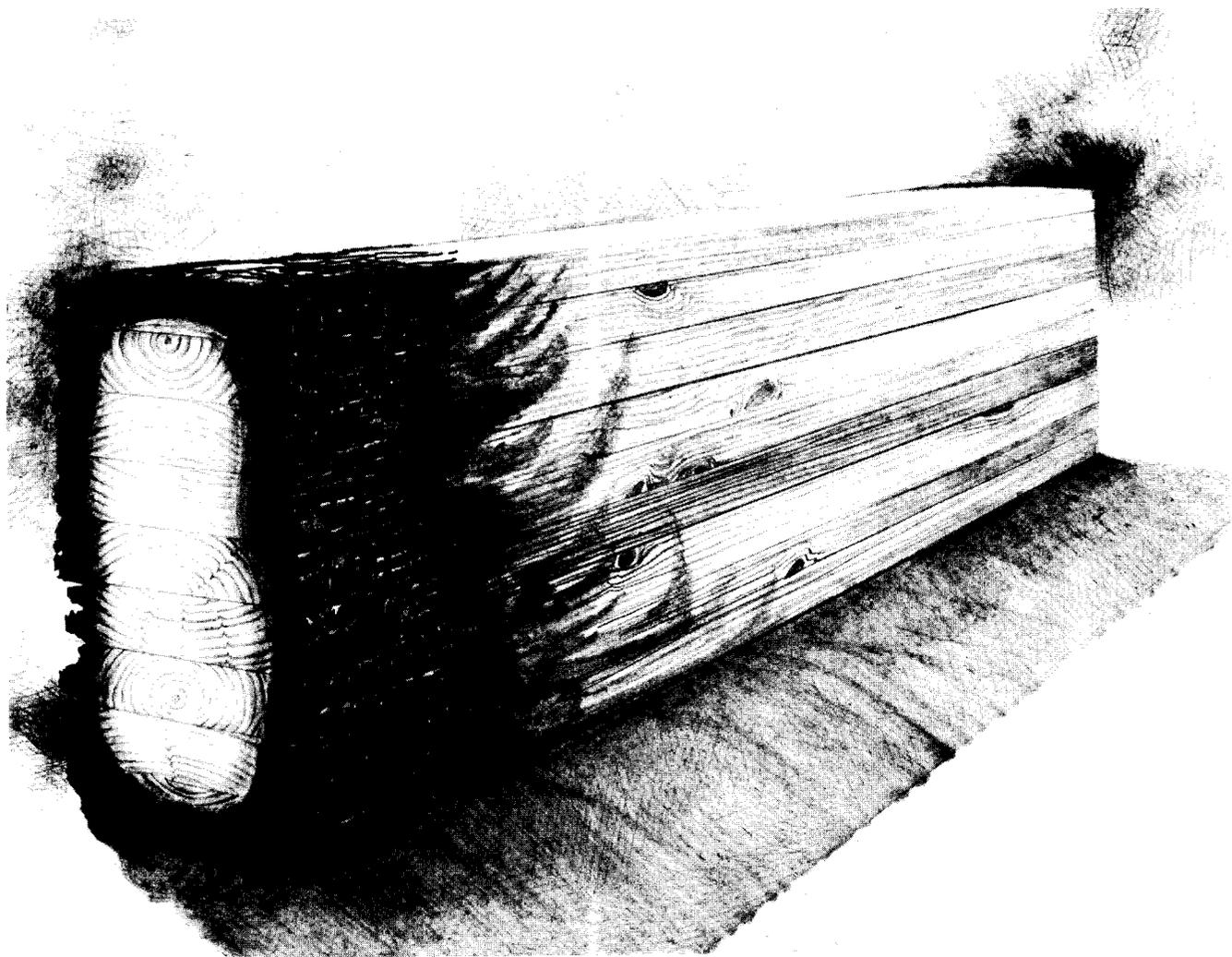
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Strength Validation and Fire Endurance of Glued-Laminated Timber Beams

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Abstract

A previous paper presented a reliability-based model to predict the strength of glued-laminated timber beams at both room temperature and during fire exposure. This Monte Carlo simulation procedure generates strength and fire endurance (time-to-failure, TTF) data for glued-laminated beams that allow assessment of mean strength and TTF as well as their variability. This paper reports an effort to validate model predictive capability through an independently fabricated set of 21 glued-laminated beams. Based upon the available data for the model input parameters on lumber strength and stiffness, finger-joint strength, and length of laminating lumber between sequential finger joints, the model of beam strength appears acceptable and possibly slightly conservative.

Refinements in the beam strength model allow its use for predicting fire endurance. In this case, the fire endurance is measured by the TTF and is defined as the time the beam will support its design load while subjected to fire. The residual strength of the beam is analytically calculated by removing the char layer, plus a finite thickness of weakened wood, from the beam cross section as fire exposure time increases.

Employing the input parameters for values of finger-joint strength and lamination grades of Douglas-fir, the fire endurance TTF was analyzed for a 5.12- by 16.50-inch 11-lamination Douglas Fir—Larch beam (24F-V4) carrying full allowable uniform load (47.7 lb/in.). (Three-sided fire exposure was assumed; however, four-sided exposure can also be accommodated.)

A simulated random fabrication and analysis of the TTF under fire exposure for 100 beams was performed. The mean TTF was estimated as 35.2 minutes with a coefficient of variation of 13.7 percent. Lateral torsional buckling was never the cause of failure in any of the simulations.

The results compared well (within a 65 pct confidence band) with the observations and predictions for timber beams reported by sources in other countries. A simulation for a single glulam beam test in cooperation with the National Forest Products Association was also conducted which predicted the result exactly.

Keywords: Glulam, beams, fire endurance, strength, model, reliability, testing.

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Strength Validation and Fire Endurance of Glued-Laminated Timber Beams

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Introduction

Another paper by the authors (Bender and others 1985) discussed the formulation of a reliability-based model to predict the strength of glued-laminated timber beams at room temperature conditions and during fire exposure. The model consists of a Monte Carlo simulation of beam fabrication and strength estimation calibrated to the strength and fire endurance of previously reported glued-laminated beam test results.

Reported here is a validation of model predictive capability using an independent set of glued-laminated beams especially fabricated for this purpose.

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Validation Beam Strength Experiments

Experimental Procedures and Results

Experiment design. –The test beam combination chosen to verify the reliability-based model was the Douglas Fir—Larch combination, 24F-V4, in American Institute of Timber Construction AITC 117 (AITC 1982). Douglas Fir—Larch was chosen because the data base was most complete for those species. The 24F-V4 combination was chosen because it was used in a study of shallow glulam beams by Marx and Moody (1981) that provided some guidelines for predicting strength of beams at room temperature.

Three beam sizes were chosen (fig. 1): The first group (type A) represents room temperature conditions with a full cross section of 5.12 by 16.50 inches. The second group (type B) simulates a 30-minute fire exposure of three sides of the beam resulting in a char depth of 0.75 inch and a cross section of 3.62 by 15.75 inches. The third group (type C) simulates a 60-minute three-sided fire exposure resulting in a char depth of 1.5 inches and a cross section of 2.12 by 15.0 inches,

A total sample size of 21 beams was selected, with seven beams in each of three groups, to allow the significant detection of a 15 percent difference between means with 95 percent confidence. The coefficient of variation (COV) for each group was assumed to be 17.5 percent. More specimens were not included because doubling the number of specimens per group would have only allowed detection of about a 10 percent difference in mean strengths and would have significantly increased the cost.

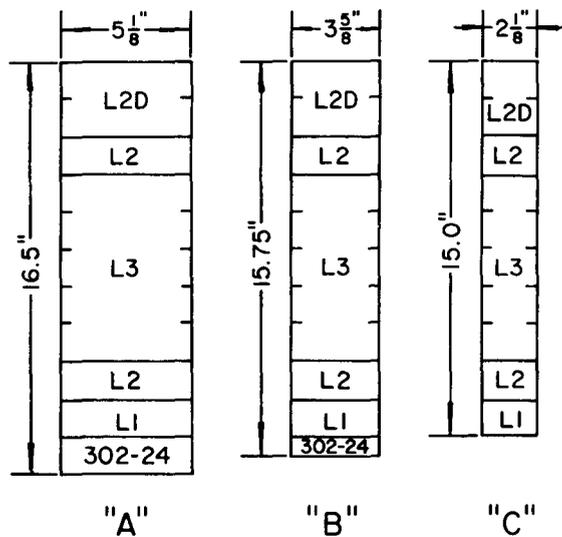


Figure 1.—Douglas Fir—Larch test beams. There were seven beams in each of these three test groups. (M151 353)

In general, the following properties were obtained during the fabrication and testing of the beams:

- Moisture content, weight, and modulus of elasticity (MOE) for each piece of lumber.
- Knots and grain deviations in the L1 and 302-24 laminations.
- The tensile strength of 20 finger-jointed laminations; 10 of L1 and 10 of 302-24 grade.
- Static bending strength modulus of rupture (MOR), beam MOE, and rupture locations of tested beams.

Further details of lumber selection and evaluation, end jointing of lumber for beam laminations, and fabrication of the beams are given in the appendix.

Beam strength properties.—The results of the static bending tests are summarized in table 1. Individual beam test results are given in the appendix. MOR values were calculated using maximum moment and gross section moduli and are unadjusted values.

End-joint tension tests.—The results of the end-joint tension tests are summarized as follows:

Sixteen of the twenty specimens tested in tension failed at least partially at an end joint. One-half of the failed joints were in L1 material and the other half in 302-24 material. Those 16 specimens had a mean unadjusted tensile strength of 4,940 lb/in.² and a COV of 20.3 percent. To provide a further indication of joint quality, two of the joints failed below 3,500 lb/in.², six between 4,000 and 5,000 lb/in.², and only one failed above 6,000 lb/in.². The tensile strengths of the 16 specimens ranged from 3,400 to 7,280 lb/in.².

Table 1.—Means and coefficients of variation (COV) for experimental and predicted modulus of rupture strengths for three glulam beam sets

Beam type	Experimental		Predicted ¹			
	Mean ²	COV	“L” set		“S” set	
			Mean	COV	Mean	COV
	Lb/in. ²	Pct	Lb/in. ²	Pct	Lb/in. ²	Pct
A	5,980	22.0	6,320	13.5	5,120	21.4
B	6,080	27.9	5,520	12.7	4,580	20.0
C	5,630	13.9	4,890	13.0	4,670	14.8

¹“L” set: Employing finger-joint strength data from estimated whole population properties.

²“S” set: Employing finger-joint strength data estimated for beams fabricated.

²Unadjusted means.

Comparison of Experimental Beam Groups

Before, during, and after testing, it became obvious that there was some problem with the end joints in the test beams and that this would influence the results. It appeared that the horizontal end joints had been “cocked” during joining as the exaggerated illustration in figure 2 shows. This left gaps between the fingers on one side of the narrow face, indicating inadequate bonding, but not on the other face. This produces a strength gradient from one face to the other. Unfortunately, these cocked joints also resulted in different end-joint tensile strength distributions, and hence different beam strength distributions. This occurred because the inadequately bonded portion (low-strength area) was increasingly planed from the A to the B to the C type beams. The conclusion reached was that the beam types A and B had significantly different end-joint distributions than did beam type C. This concession can be substantiated by four observations: (1) visual inspection before testing, (2) visual inspection after testing, (3) failure characteristics, and (4) strength results. These are discussed further in the following paragraphs.

Visual inspection before testing.—Gaps between fingers were visible on one narrow face, while the fingers on the other narrow face were pressed tightly together for both type A and B beams. No gaps were visible for the type C beams. It appeared as if the type A beams contained almost all of the inadequately bonded portion of the joints and the type B beams still contained some of the inadequately bonded portion of the joints. By the time 1.5 inches were planed off of both sides for the type C beams, nearly all of the inadequately bonded portion of the joint was removed, and therefore the end-joint strength was more typical.

Visual inspection after testing.— There was evidence of low percentages of wood failure after testing on the edge of the each joint where the gaps had been observed for beam types A and B, but not for beam type C.

Failure characteristics.— The majority of beams failed in the tension lamination, as expected. But the majority of type A and B beams failed at an end joint in the tension lamination (see appendix), while the majority of type C beams failed at a knot in the tension lamination. Only one type C beam failed at an end joint in the tension lamination. Yet, all three beam types had finger joints subjected to about the same percentage of maximum moment.

Strength results.— The means and COV's for the three beam types are summarized in table 1. An analysis of variance revealed no significant differences among the three average MOR's of the beam types. Based on previous research and experience, type A, B, and C beams were expected to have MOR's of 6,500 lb/in.², 6,000 lb/in.², and 5,500 lb/in.², respectively; and significant differences should be detectable among the beam groups. Marx and Moody (1981) found about a 15 percent difference in strength between shallow beams with specially graded tension laminations and beams with regular laminating grades for tension laminations. Fox (1978) tested some deeper beams without specially graded tension laminations and found them to be about 25 percent below their proposed design stress. Thus, a difference in strength of at least 15 percent between beam groups A and C was expected, just due to loss of the specially graded 302-24 tension laminations. Instead, there was less than a 6 percent difference between the beam types fabricated. The actual average strength of beam type A (5,980 lb/in.², should have been significantly higher (Marx and Moody 1981, 1982). The actual strengths of beam types B and C were those expected from previous work.

A 17.5 percent COV was expected for the three beam types. The COV's for beam types A and B (22.0 and 27.9 pct) were higher than for beam type C (13.9 pct). Furthermore, the COV's for beam types A and B were higher, and that for type C lower, than expected, based on previous research and experience.

Summary.— Overall analyses and observations indicated a difference in beam types A and B as compared with beam type C. That difference was due to end-joint quality. Beam types A and B had end joints with below-average strength distributions, while beam type C had end joints with a more typical strength distribution. These observations will be seen to have had a significant influence on the validation of model predictive capability.

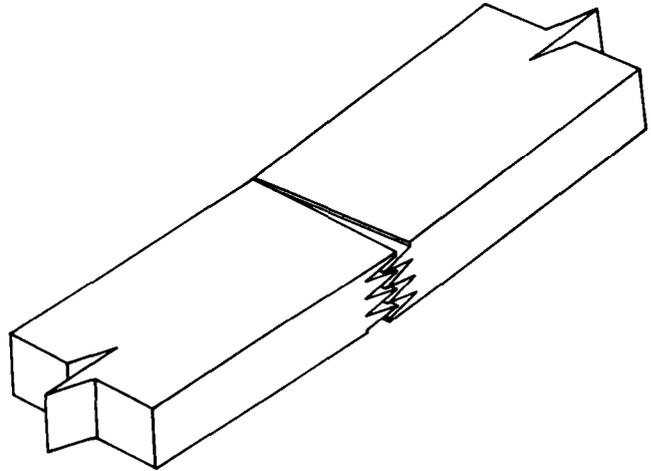


Figure 2.—Exaggerated illustration of defect observed in some finger joints employed in fabricated beams. Joint is "cocked" in the wide dimension plane. (M151 352)

Bending Strength Predictions

As will be described in more detail shortly, the prediction model requires the input of an appropriate tensile strength distribution (or end joints). Thus, due to the two different distributions of end joints in the test beams, it was necessary to make two sets of predictions.

The first set was based on an end-joint tensile strength distribution developed from approximately 200 industry tension tests of well-made end joints from numerous laminating plants. This end-joint set will be referred to as set “L,” referring to a reasonably “large” sample size.

The second set of predictions was based on an end-joint tensile strength distribution developed from the 16 end-jointed tensile test specimens randomly selected from beam lamination lumber described earlier. This end-joint set will be referred to as set “S,” referring to a “small” sample size.

Statistical analysis revealed that the estimated Weibull distribution for 16 end-jointed specimens (set “S”) could not be represented by the predicted Weibull distribution for the industry-developed joint strength “L” set.

The procedure used in the developed model to make the predictions is described next.

Procedure

The rupture strengths of the three sets of beams were estimated using a reliability-based technique (Bender 1980, Bender and others 1985). Required inputs to the procedure are grade order, species of the laminations and lumber lengths in the beam, distributions of MOE and tensile strength, and distributions of end-joint tensile strength.

This model uses Monte Carlo simulation to assemble glulam beams in the computer. For each of these beams the gross MOE, ultimate moment, apparent MOR, failure location, and failure mode are calculated. The following steps provide a summary of the simulation procedure for a single beam. These steps are repeated according to the sample size.

1. Laminating lumber of random length, MOE, and tensile strength is dealt into the bottom layer of the beam until the length requirements have been satisfied. Each subsequent layer is generated according to the layout of the beam by grade and species.

For both sets of predictions, the same distributions for MOE were determined based on the data collected on the lumber used to build the test beams. Corresponding tensile strength values for the MOE-tensile strength (E-T) pairs were calculated based on regression equations determined from earlier data and described by Bender (1980) and Bender and others (1985).

2. A random end-joint tensile strength is assigned wherever two pieces of lumber meet end-to-end. End joints in the computer assembly are not allowed to occur within 6 inches of each other on adjacent tension laminations as specified by Voluntary Product Standard PS-56 (AITC 1973).

3. Steps 1 and 2 are repeated until the entire beam has been assembled. All of the random strengths and stiffnesses are recorded in arrays, along with the location of each end joint.

4. A transformed section analysis (Brown and Suddarth 1977) is repeated across the entire beam at a specified increment of beam length. In each case, tensile strength and stiffness are used to calculate the ultimate moment. Then, the ultimate moment, gross MOE, failure location, and failure mode are stored in an array.

5. A transformed section analysis is performed at each end-joint location. End-joint tensile strength and stiffness are used to calculate the ultimate moment. The end-joint stiffness is taken to be the average stiffness of the two connecting pieces of lumber.

6. The minimum value of the ultimate moments of steps 4 and 5 is recorded. This value defines the ultimate moment carrying capacity for the assembled glulam beam. These moments are compensated for the location on the moment diagram for two-point loading.

7. The apparent MOR is calculated by assuming a homogeneous beam cross section.

8. The ultimate moment carrying capacity of the beam and the associated MOR, gross MOE, failure location, and failure mode are recorded.

The two fundamental assumptions of the model are that:

1. Beam bending strength is limited by the tensile strength of the laminations or end joints. Failure occurs when the tensile strength parallel to grain of a lamination is exceeded, with the stress for each lamination being calculated at the center of the lamination. This is referred to as “first failure.” Gradual-type failures which result in redistribution of stresses are not predicted by this model.

2. Beams behave elastically until first failure is detected.

Bending strength predictions were made for all three beam groups using the “L” or “S” set end-joint strengths. For the beams with reduced cross sections, the ultimate moment carrying capacity was first calculated. The MOR was then computed using the reduced cross section and the usual flexural formula.

The simulated fabrication and simulated bending test were developed with 500 randomly simulated beams per beam group. A summary of the input parameters used in the predictive model is given in table 2.

To fully simulate the behavior of beams actually tested, a correction must be made for the load conditions. The span between supports was 25.5 feet and between the two load points 5 feet for all of the experimentally tested beams. The 5-foot separation between the load points is essential to transforming a known tensile strength, T, of a long piece of lumber to one, shorter in length (T'). This transformation was given previously by Bender and others (1985).

$$T' = T(N)^{1/\eta} + \mu [1 - (N)^{1/\eta}] \quad (1)$$

Table 2.—The input parameters used to validate the beam strength prediction model include Weibull parameters for modulus of elasticity data, regression parameters which relate tensile strength to modulus of elasticity, and log-normal Parameters used to define the length of the Pieces of lumber. Modulus of elasticity Weibull scale and location parameters (σ and μ) must be multiplied by 1 million. Also included are Weibull parameters for the tensile strength of horizontal finger joints (“L” and “S” sets). Tensile strength Weibull scale and location parameters (σ and μ) must be multiplied by 1,000

	Weibull parameters ¹			Regression parameters ²			Length parameters ³		
	Sample size	Scale (σ)	Shape (η)	Location (μ)	B ₀	B ₁	K	λ	ξ
		<i>Lblin.²</i>		<i>Lblin.²</i>					
MODULUS OF ELASTICITY									
302-24	40	0.89	2.44	1.82	7.71	0.306E-06	0.325E-07	2.283	0.198
L1	55	.72	2.47	1.69	6.83	.561E-06	.298E-07	2.283	.198
L2D	79	1.05	3.71	1.18	6.58	.678E-06	.231E-07	2.471	.166
L2	74	1.03	4.54	.99	6.58	.678E-06	.231E-07	2.471	.166
L3	156	.97	3.91	.78	6.88	.470E-06	.336E-07	2.471	.166
TENSILE STRENGTH									
“L” set finger joints		3.81	3.55	3.35	--	--	--	--	--
“S” set finger joints		2.18	2.09	3.01	--	--	--	--	--

¹The Weibull parameters were determined based on the data collected on the lumber used to build the experimental beams.

²The regression parameters were determined from earlier E-T paired data and described by Bender (1980) and Bender and others (1985).

³Assumed shorter lumber lengths on the average for the higher quality grades.

The parameters η and μ are the shape and location parameters for the three-parameter Weibull distribution of tensile strength. The value N equal to 2.4 is computed by dividing the distance between grips of the experimentally tested tensile specimens used to develop E-T regression parameters (12 ft) by the distance between the two points of load application of the test beams (5 ft). Further discussion of how this transformation was derived is given in a previous report (Bender and others 1985).

Results of Simulations and Tests

The results of the beam simulations as portrayed by the three-parameter Weibull distribution parameters and mean and COV are shown in tables 3 and 4. As mentioned, for purposes of easy identification of the end-joint input data used, the first set is termed “L” to represent end-joint strengths for a large population of industry Douglas-fir end joints, and the second set is termed “S” to represent a small set of finger joints collected at the same time the experimental beams were fabricated.

Table 3.—Parameters for three-parameter Weibull distribution of predicted beam strengths—two different assumptions of finger-joint strength distributions are employed in the predictive model—set “L” and set “S”

Beam type	Set “L” ¹			Set “S” ¹		
	Scale (σ)	Shape (η)	Location (μ)	Scale (σ)	Shape (η)	Location (μ)
	<i>Lblin.²</i>		<i>Lblin.²</i>	<i>Lblin.²</i>		<i>Lblin.²</i>
A	3,020	3.53	3,590	2,650	2.28	2,770
B	2,660	3.78	3,120	2,460	2.58	2,390
C	2,150	3.29	2,960	2,240	3.17	2,660

¹“L” set: Employing finger-joint strength data from estimated whole population properties. “S” set: Employing finger-joint strength data estimated for beams fabricated.

Table 4.—Proportion of finger-joint location failures controlling simulated beam strengths

Beam type	“L” set finger joints	“S” set finger joints
	<i>Pct</i>	<i>Pct</i>
A	44	85
B	35	81
C	4	30

¹Based on 500 simulations for each of the six groups in the table.

Figure 3 shows a typical histogram of the predicted MOR for beam group A having end joints of the "S" joint set. An overlay of the corresponding three-parameter Weibull density functions is included in the figure. A typical histogram and the corresponding three-parameter Weibull density function of the predicted MOR for beam group A having finger joints from the "L" set are of similar characteristics.

Experimental beam results.— Figure 4 shows the mean MOR's for the simulated and experimentally observed three beam types. Note that the observed values are greater than those predicted, using either the "L" or "S" group joint strength, for beam types B and C. For type A, the mean strength falls between set "L" and "S" predictions, but is closer to set "L". It can also be observed that the experimental results fall well within the 90 percent confidence intervals for the "L" set predicted means.

As stated earlier, 21 beams (7 beams in each of the 3 groups, A, B, and C) were fabricated with the expectation of being able to detect a 15 percent difference between types with 95 percent confidence. This was based upon results obtained in previous research by Marx and Moody (1981). In this study, however, an analysis of variance did not detect any significant difference in mean strength between fabricated type A and type B beams, but did detect about a 6 percent difference between A (or B) and type C beams. This is well below the 15 percent difference anticipated.

Correlation to Experimental Beams⁴

Although several statistical comparisons of the data were made, only one technique was used to compare the observed beam strength behavior with that simulated using the two finger-joint strength sets: This was a comparison of cumulative density plots of all data. Other statistical analyses generated inconclusive results.

Cumulative density graphs for the observed and simulated beam strengths of each beam type are shown in figure 5. Note how sensitive the shape of the cumulative curve for the observed beam strengths is to each of the seven values obtained. For the type A and C beams (fig. 5), the observed curve parallels the simulated curves quite well (except for the last observed strength value in type A). The shape of the observed curve for type B is nonparallel for nearly all points.

If the simulated curves truly represent the distribution in strengths to be expected with a very large number of beams, then the observed strengths for the beams constructed are not necessarily representative of what can be expected because of the relatively small number tested. If high and low levels of observed strength reflect much higher or lower levels of strength in the true cumulative density curve for a much larger population, these outlier values not only warp the curve shape, but have an exaggerated influence on the mean strength. This, of course, negates conducting meaningful analysis of variance.

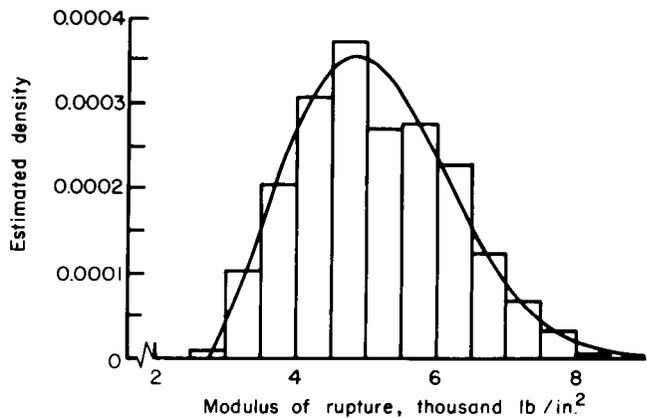


Figure 3.—Histogram and estimated three-parameter Weibull density functions of modulus of rupture for simulated type A Douglas-fir beam (5-1/8- by 16-inch cross section). Finger-joint distributions used as input was the "S" set. The histogram and density function for the "L" set are similar. (ML85 5371)

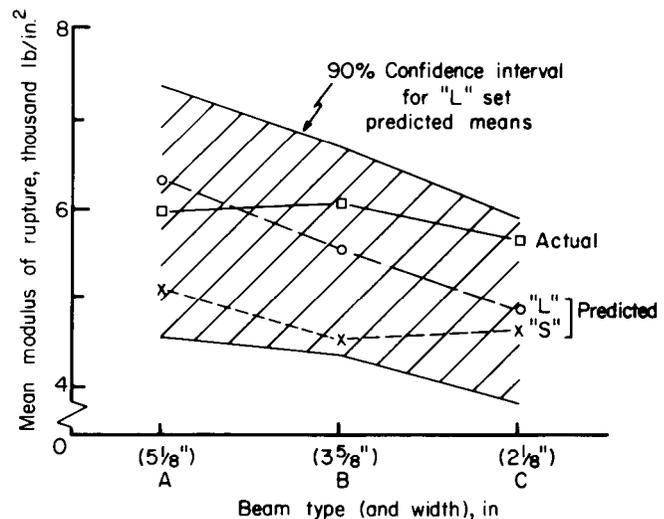


Figure 4.—Predicted and actual mean strengths as a function of beam type (and width of beam). (ML85 5372)

⁴Special appreciation is expressed to Dr. James W. Evans, Mathematical Statistician, Forest Products Laboratory, for his advice and assistance.

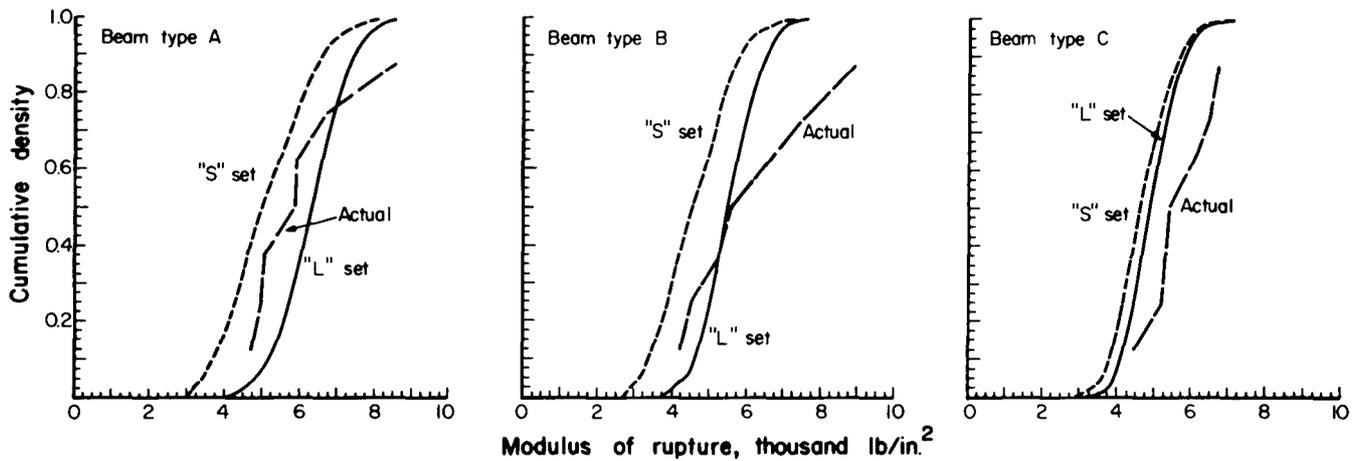


Figure 5.—Cumulative distribution function graph of observed and simulated beam strengths. (ML85 5373)

Discussion

It is well known that both lamination and finger-joint quality are the key factors controlling glued-laminated beam strength. Our results support that observation. It is clear from the correlation of beam strength experimental results to those predicted that predictive capability is sensitive to both the lamination strength and strength of the finger joints.

We feel the results shown in figure 4 for the fabricated beams exhibit a mean strength typical of well-made finger joints for type C beams, but somewhat less than expected for type B and much less than expected for type A (Marx and Moody 1981, 1982). The strengths of actual beams tested fall well within a 90 percent confidence interval for the "L" set predicted mean strengths. If, in addition, the points for the mean strengths of the type A and type B beams were adjusted upward proportionately, the line through the points would likely parallel and be greater than that predicted using the "L" set of finger-joint strength data. The same cannot be said for the strengths predicted by "S" set finger-joint data, as it is assumed the strength of finger joints per unit area remains constant with reduction in lamination width.

In this case, the factor reducing the prediction capability was the "horizontally cocked" appearance and subsequent lower strength of finger joints. The "cocked" appearance was particularly apparent in the type A beams. This influence can be explained to reflect favorably on the predictive capability of the strength model. The full-size type A beams should be accurately simulated by the model if finger-joint strengths representative of those used in the test beams are employed. However, if the behavior is assumed to apply to reduced-size beams, a major error can be introduced. Recall that the beams were planed to requisite width after fabrication, removing the widest gapped material at the "cocked" finger-joint locations. This should increase the mean tensile strength per unit area of the residual section of the laminate.

The finger-joint strength effect is clearly seen in figure 5, where the frequency of failure at finger-joint locations is given for each simulated beam type. The finger-joint strengths account for the preponderance of achieved strengths in the "S" set type A and type B beams. The "L" set beams, however, show the more balanced response consistent with experience; that is, they have the number of failures of the finger joints nearly equivalent to that of the lamination stock. The type C beam failures are largely shown to be attributed to the weaker laminating stock above the tension lamination. Hence, one can state with some confidence that the "L" set finger-joint strength data approximate the experimental strength of all beam types after rationally taking into account the aberrations in finger-joint strength. One can conclude that the developed model should be a conservative and sufficiently accurate predictor of the strength and variability for glued-laminated beams.

Beam Fire Endurance Prediction

General

A fire endurance prediction model was developed by making minor refinements in the beam strength model. This was described in a previous paper by Bender and others (1985). The fire endurance of glulam beams is measured by the time-to-failure (TTF), where TTF is defined as the length of time that a structure will support its design load when subjected to intense fire conditions. The fire endurance model can be used to predict the distribution of TTF for any glulam beam of interest. Fire is simulated by removing the char layer from the beam cross section. The thickness of the char layer R is given by

$$R = \beta t + \delta \quad (2)$$

where

β = char rate

t = fire exposure time

δ = finite thickness of residual wood which is weakened by the elevated temperature and moisture.

Two of the assumptions of this model are that char rate β and residual thickness δ remain constant. These two assumptions have received considerable experimental support (Schaffer 1965). Another assumption is that the unit strength and E properties of individual laminations remain constant as the cross section is reduced.

The following steps summarize the simulation procedure of the fire endurance model for a single beam. These steps are repeated for each beam in the simulation:

1. A beam is randomly fabricated in a computer simulation in the same manner as that for strength assessment.
2. Full design load (that which develops the full allowable stress in bending) is applied. Allowable stress is compensated for beam size.
3. A transformed section analysis is performed along the entire length of the beam at specified increments of length and at each finger-joint location.
4. Computed stress levels are compared with corresponding tensile strength values. If tensile strength is exceeded, failure occurs and TTF is recorded.
5. Critical moment permitted by lateral torsional buckling is calculated. If the critical moment is exceeded by the applied moment, failure occurs and the TTF noted.
6. Time is increased 1 minute, and the corresponding char thickness as produced by a standard fire exposure (American Society for Testing and Materials Standard E 119 (ASTM 1979)) is removed from the cross section.
7. Steps 3 through 6 are repeated until beam failure occurs.
8. Steps 1 through 7 are repeated for each beam simulated.

Critical Parameters

The parameters critical to predicting room temperature beam strength have already been discussed by Bender and others (1985). There is, however, a need to explain the origins of parameters necessary to the analysis of strength under fire exposure. These are:

1. Conversion of the beam span, L , under two-point concentrated load to an effective span, l^* , under uniform load common in fire exposure tests.
2. The depth of char as a function of char rate, β , and time, t .
3. Compensation for the loss of strength of the heated uncharred wood by assuming an additional layer of wood, δ , has zero strength.

Effective span, l^* .—The predictive model for beam strength and fire endurance is sensitive to whether the load is applied uniformly or concentrated at several points. To consistently compensate for this difference one should convert the span under concentrated load, L , to an effective length, l^* , under uniform load. For beams tested under two-point loading, a shear span to beam depth (d) ratio of at least 15:1 is recommended to minimize the likelihood of failure in horizontal shear. As a result, the effective span length, l^* , for a uniformly loaded beam was defined as:

$$l^* = L - 15d \quad (3)$$

This effective span is further employed in a transformation equation (1) to determine the expected tensile strength of the laminating lumber between finger joints as compared with that determined for experimentally tested lumber of fixed length.

Char rate, β .—The char rate is assumed to be an average of 0.025 inch/minute for Douglas-fir. Though this char rate could be entered as a random variable as well, it was not in this investigation.

Zero-strength layer, $\beta t + \delta$.—The char layer forms at a steady rate, β , under standard fire exposure. Because it is highly porous and fissured, it can be assumed to have no load-carrying ability. The wood below the char layer is heated, however, and moisture moves through it as it dries. Both temperature and moisture content affect wood strength and stiffness. For Douglas-fir, in large sections such as glued-laminated beams and columns, the temperature achieves a quasi-steady-state distribution below the char-wood interface (Schaffer 1977). The moisture gradient has also been experimentally investigated for Douglas-fir at 12 percent moisture content by White and Schaffer (1981). Procedures are available to compensate wood strength and MOE for temperature and moisture content (Schaffer 1984). If the tensile, compressive, and MOE values are computed for uncharred wood in a fire-exposed section, the response is as shown in figure 6 (Schaffer 1984). These properties are most affected within a 1.5-inch (40-mm) layer below the char-wood interface in a fire-exposed section.

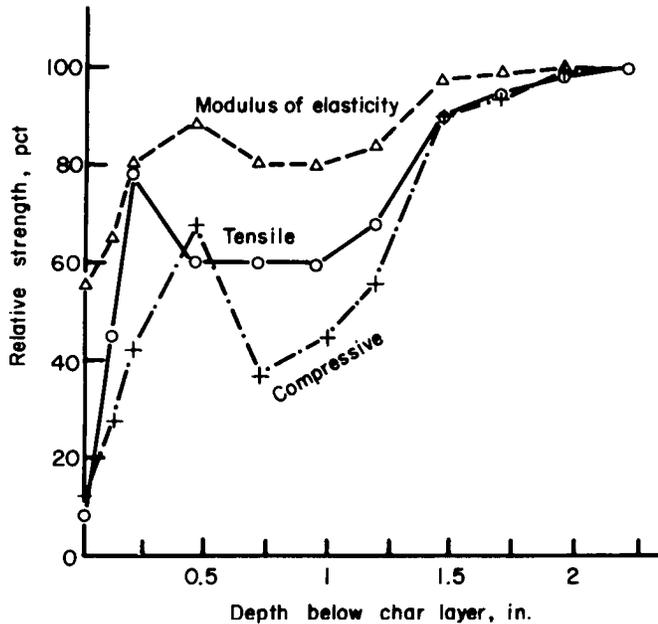


Figure 6.—Relative modulus of elasticity and compressive and tensile strength as a function of distance below char layer in softwood section under fire exposure. (Expressed in percent of that at 25 °C and initial moisture content of 12 pct.) Duration of fire exposure should be equal to or greater than 20 minutes to apply results of this figure. (ML85 5374)

To simplify analysis of beam fire endurance, we determined whether a layer of uncharred wood of thickness, δ , for which negligible strength is assumed, can be subtracted from the beam cross-sectional dimensions in addition to the char layer thickness, βt . This was done by (a) averaging the tensile, compressive, and MOE response over the 1.5-inch (40-mm) heated layer, (b) analyzing the beam using transformed section analysis, and (c) ascribing the full loss of strength to a heated layer of thickness (δ).

The mean strength properties and variation for the 1.5-inch-thick (40-mm) heated layer are expressed as percentages of those at room temperature with wood moisture content of 12 percent:

	Mean	COV
Tensile strength	66.1 pct	14.5 pct
Compressive strength	54.4 pct	19.3 pct
MOE	83.4 pct	9.0 pct

Analyzing a three-sided, simulated, fire-exposed beam for strength using transformed section analysis resulted in the observation that the effective mean tensile strength is 79.3 percent of that at room temperature. Using this information, it was estimated that a layer (δ) 0.3-inch-thick can be deducted from the heated zone beneath the char on fire-exposed sides of the beam and that the residual strength with duration of fire exposure can be estimated (fig. 7). The total zero-strength layer becomes $\beta t + \delta$.

Analysis

Employing the “L” set parameters for values of finger-joint strength and typical properties of lamination grades of Douglas-fir, the fire endurance TTF was analyzed for 5.12- by 16.50-inch 11-lamination Douglas Fir—Larch beams (24F-V4) of 25.5-foot span and carrying full allowable uniform load (47.7 lb/in.). Three-sided fire exposure was assumed; however, four-sided exposure can also be accommodated.

The simulated random fabrication and analysis of the TTF under fire exposure for 100 beams was performed. The mean TTF was estimated as 35.2 minutes with a COV of 13.7 percent. Lateral torsional buckling was never the cause of failure in any of the repeated simulations. If the bottom lamination is defined as the first lamination, failure normally initiated in the second lamination (57/100), but was found to occur in the first (25/100) and third (18/100) as well. The charring of about half the thickness of the first lamination contributed to and explains much of this result.

The TTF for three-sided fire exposure of beams of the same dimension and subjected to full design load was also calculated using two other methods by Lie (1977) and Meyer-Ottens (1979). Both assume that a model applies to the form

$$M/S(t) = \alpha \sigma_{cr}$$

where

M = the maximum applied moment

$S(t)$ = the section modulus of the uncharred wood section as a function of duration, t , of fire exposure

σ_{cr} = strength of wood at room temperature

α = the ratio of the strength of the outer uncharred wood fibers of the beam to that at room temperature.

The factor, α , varies from 0.5 to 0.8 in international literature.

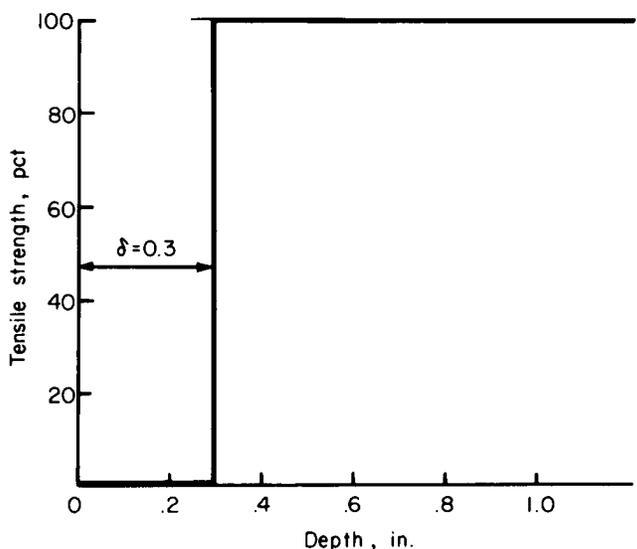


Figure 7.—Simplified diagram attributing all tensile strength loss in a charring beam to a thickness of 0.3 inch into the heated wood below the advancing char. (ML85 5375)

The TTF prediction obtained using Lie (1977) was 33.2 minutes, and that employing Meyer-Ottens (1979) was about 30 minutes. A more refined analysis developed by A. Haksever⁵ that expands upon Meyer-Ottens (1979) indicated a TTF of 31.2 minutes.

The above results are within the 65 percent confidence band (30.4 -40.0 rein) of the model for fire endurance presented in this paper. Hence, the results are similar. However, only the results of Meyer-Ottens (1979) and Haksever⁵ have a strong experimental fire endurance data base. A total of 35 glulam beams were tested under load and fire exposure. Unfortunately, the design stress statistics are not reliably known so that the applied load can be compared directly with those allowed in North American practice. It appears, however, that West Germany applies a smaller general adjustment factor on the 5 percent exclusion limit for MOE.⁶ In North America the factor is 2.1, and in Germany, evidently, it is 1.88. In essence, then, allowable design stresses are 11.7 percent higher in Germany than in North America. An 11.7 percent decrease in developed beam stress can increase the TTF of a 5.12- by 16.50-inch glulam beam ($S = 3,810 \text{ cm}^3$) about 7 minutes (Meyer-Ottens 1979). This results in a North American fire endurance estimate of 38.2 minutes.

The TTF of 35.2 minutes predicted by the model in this paper is seen to fall between the 33.2 minutes and 38.2 minutes estimated from these sources and observed to be well within the confidence limit of ± 4.8 minutes obtained by the simulation model.

The United States and Canada recently conducted a single glulam beam fire endurance test. The 11-lamination Douglas Fir—Larch beam (24 F-V4, 10-lamination design plus additional 302-24 grade tension lamination) was 8.69 by 16.44 inches in cross section (fig. 8). The beam span was 16.97 feet from center to center of bearing. It was loaded to 71.5 percent of full design load. The adjusted full allowable bending stress for this section was 2,396.5 lb/in.² and the load was applied at the three quarter points.

The beam was exposed to ASTM E 119 (1979) fire exposure on three sides. Failure to carry the load (rupture) occurred at 86.25 minutes.

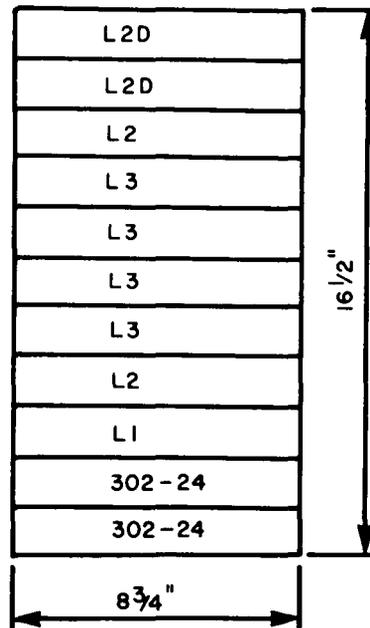


Figure 8.—Layup of Douglas Fir—Larch glulam beam experimentally fire endurance tested. (ML85 5376)

Prior to testing, the TTF was simulated, using the model developed here, for 100 randomly generated beams. Fifty were subjected to full design load and 50 to 71.5 percent of full design load. The calculated mean TTF and standard deviation were:

	Full load (min)	71.5 percent load (min)
Mean TTF	70.5	86.0
Standard deviation	10.2	7.5

The comparison of predicted to observed TTF shows that the experimental result coincides with the predicted, and that a typical beam fully loaded could expect to have a TTF of 70.5 minutes. In addition, 95 percent of the beams so tested under full load would be expected to have a TTF greater than 53.7 minutes!

⁵Haksever, A. Private communication (12/80). Institute für Baustoffe, Massivbau und Brandschutz. Braunschweig, West Germany: Technische Universität Braunschweig, 1980.

⁶Goodman, James. Private communication (7/21/80). Fort Collins, Co: Colorado State University, Department of Civil Engineering; 1980.

Conclusions

Summary

Simulating the reduction in beam cross section due to charring and assuming an additional layer of thickness (0.3 in.) has zero strength allows the TTF under fire exposure to be estimated in a straightforward manner.

Prediction of beam strength for one beam type of 5.12- by 16.50-inch cross section compares favorably with other existing available analyses and experiments.

Even though the beam had an initial depth-to-breadth ratio of 3:2 and initial span-to-depth ratio of 18:5, predicted load-carrying capacity under fire exposure was due to exceeding the critical MOR rather than lateral buckling in all simulations.

Full validation of the fire endurance model is obstructed by the unavailability of a fire endurance data base directly translatable to North American practice. More independent experiments to test the adequacy of the beam simulation model for fire endurance are needed.

A reliability-based computer model employing a Monte Carlo simulation technique was previously developed to predict the variation in strength and fire endurance of glulam beams (Bender 1980, Bender and others 1985). One may conclude from the validation effort reported here that:

1. The model is a slightly conservative predictor (underestimator) of beam strength as based upon the rational analysis of independent test results.
2. An 11-lamination Douglas-fir glulam beam of 5.12- by 16.50-inch cross section and of 24F-V4 design is predicted to have a time to failure under standard fire exposure (ASTM E 119-79) of 35 minutes (COV of 13.7 pct) while carrying full design load. This compares well with foreign literature.
3. The model accurately predicted the results of one well-controlled full-size glulam beam fire endurance test. The TTF under 71.5 percent of full design load was predicted to have a mean of 86.0 minutes (COV of 7.5 pct), and the actual beam failed at 86.25 minutes. (Under full design load, the model estimated mean TTF at 70.5 minutes.)
4. The simulation model provides predicted mean TTF estimates for beam types as well as the COV. Information of this type can be used in a second-moment reliability analysis for fire safety if variation in the fire exposure severity and applied load are additionally available.

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Appendix

Detailed Procedures and Results

Lumber Selection and Evaluation

The test material was selected from the stock available at the laminating plant that manufactured the test beams. It was graded according to West Coast Lumber Inspection Bureau grading rules (WCLIB 1975) by a plant grader and an American Institute of Timber Construction (AITC) representative. The selected nominal 2 by 6 Douglas-fir material ranged from 12 to 20 feet in length.

To aid in the analysis of results, moisture content, weight, and modulus of elasticity (MOE) were determined for each piece of lumber. The moisture content was determined by averaging three readings taken along the length of each lamination with a power-loss-type moisture meter. The MOE values were determined with an E-computer which used a vibration technique.

No special end-jointing order was used for the L2D, L2, and L3 material. However, the order for each piece of the L1 and 302-24 material to be end jointed was randomly assigned. No special selection criteria were imposed on the tension laminations, as had been done in several previous studies (e.g. Marx and Moody 1981, Moody 1977).

Following end jointing, individual laminations were randomly assigned to one of the three beam groups. Each set of three laminations of a particular grade was randomly assigned, one to each of the three beam groups, as encountered. It was observed that the random assignment of the L1 and 302-24 tension laminations resulted in the three beam groups having balanced characteristics of end-joint location, knot size, stiffness, and slope of grain in the midportion of the beams.

The 21 beams were next assembled into a dry layup. Laminations were occasionally reversed to meet AITC end-joint spacing requirements of 6 inches in adjacent laminations (Voluntary Product Standard, AITC 1973). In spite of that precaution, two end joints in beam A03 were spaced slightly less than 6 inches apart.

Lumber identifications and end-joint locations were recorded for all of the laminations in each beam. All 3/8-inch or larger knots were mapped between 7.0 and 20.0 feet on one face of the following laminations: (1) all of the L2D and L2 laminations, (2) the L1 laminations for the 14 beams with all or part of a 302-24 tension lamination, and (3) the L3 lamination nearest to the tension side of each beam.

Knots and grain deviations were measured on both faces of the following laminations: (1) the full length of the 7 L1 tension laminations, and (2) between 7.0 and 20.0 feet of the 14 302-24 tension laminations.

End-Joint Tension Test Specimens

Twenty 12-foot-long tension test specimens, with end joints located within the middle 8 feet, were end jointed at the same time the material for the test beams was end jointed. Ten of the specimens were 302-24 material and the other 10 were L1 material. The lumber for these tensile specimens was selected from the same stock and in the same manner as the L1 and 302-24 material for the test beams. The lumber data were also collected in the same manner except that the knots and grain deviations were not measured.

Beam Manufacture

The 21 Douglas-fir test beams were manufactured in 1980 by a commercial laminator following their normal plant procedures. The beams were certified as conforming to the Voluntary Product Standard PS-56-73 for Structural Glued Laminated Timber. A horizontal finger joint (fingers visible on the narrow face) was used with a melamine adhesive. A phenol-resorcinol adhesive was used for face gluing. Several end joints were tested in static bending while still hot before the test material was end jointed. No significant loss in strength was detected at this time.

After gluing, the group A beams were planed to 5.12 by 16.5 inches, the group B beams to 3.62 by 15.75 inches, and the group C beams to 2.12 by 15.0 inches. For all groups, equal amounts were planed from both sides of the beam. Half of the tension lamination thickness in group B beams was removed by a sanding operation.

Beam Test Procedures

The beams were tested in static bending according to American Society for Testing and Materials Standard D 198 (ASTM 1976). Figures A-1 and A-2 show the test setup for the beams. Two-point loading was used, with the span between the reactions equal to 25.5 feet and the distance between the load heads equal to 5.0 feet. This provided a shear span-to-depth ratio of approximately 15:1 to maximize bending-type failures. Figure A-2 illustrates the additional lateral support needed to test the group C beams.

An X-Y plotter was used to provide a continuous record of the machine test load versus the full-span deflection. A wire, stretched taut across the neutral axis of the beams by springs at each of the reaction points, provided a base for the deflection measurement. Two linear variable differential transformers, mounted on each side of the beam and attached to the wire, measured the deflection at the center of the beam.

The 12-foot-long end-jointed test specimens were tested in tension according to ASTM D 198 (ASTM 1976). The free span between grips was about 7.0 feet. All of the tension specimens were planed to a width of 5-1/8 inches before testing.



Figure A-1.— Test beam setup. (M150 164)

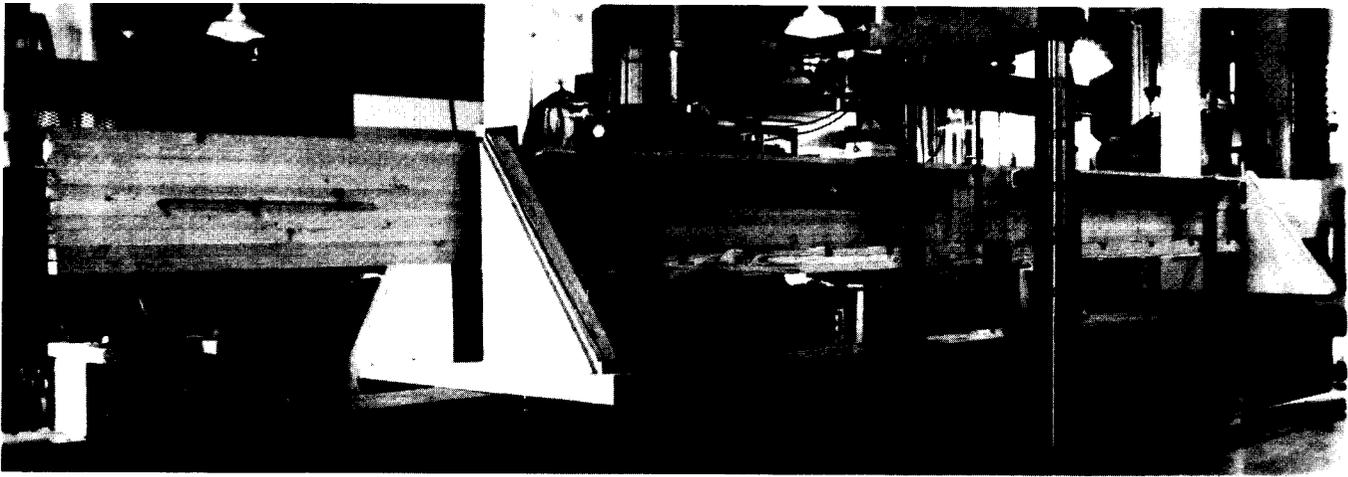


Figure A-2.— Additional lateral supports were required to test the type C beams. (M150 233-3)

Lumber Data Results and Analysis

The properties of the lumber used in the beams and the end-joint test material are summarized in table A-1.

MOE data for each of the five grades were fitted by a three-parameter Weibull distribution. The Weibull parameters were estimated numerically using the method of maximum likelihood (Steel and Torrie 1960), and are given in table 2. The sample sizes were large enough to warrant using the Kolmogorov-Smirnov (K-S) test (Birnbaum 1952) to check the relative goodness of fit of the assumed Weibull distribution. For all five groups, the K-S test indicated a good fit.

Beam Results

The moisture content, specific gravity, and MOE of each beam are shown in table A-2 along with the MOE test results. The percent of the ultimate load on the finger joint in the tension lamination and failure comments are also provided for each test beam.

Table A-1.—Summary of the average lumber properties of the test material

Lumber grade	Number of pieces	Moisture content ¹	Specific gravity ²	Modulus of elasticity ³	
				Average	Coefficient of
		<i>Pct</i>		<i>Million lb/in.²</i>	<i>Pct</i>
BEAM MATERIAL					
302-34	40	13	0.52	2.60	13.5
L1	55	12	.50	2.32	12.2
L2D	79	12	.48	2.12	13.1
L2	74	13	.44	1.93	12.2
L3	156	12	.44	1.69	16.4
PIECES FOR END-JOINTED TENSION TESTS					
302-24	10	12	.52	2.48	9.7
L1	10	12	.51	2.31	8.8

¹Determined with a power-loss-type moisture meter.

²Based on weight adjusted to oven-dry and volume at the measured moisture content.

³Determined with E-computer.

Table A-2.—Results of bending tests for three groups of seven Douglas Fir—Larch beams used to validate the simulation model

Beam number	Moisture content ¹	Specific gravity ²	Modulus of rupture	Modulus of elasticity	Load on finger joints in tension lamination ³	Failure comments ⁴		
						Tension lamination knot	Tension lamination finger joint	Other
	<i>Pct</i>		<i>Lb/in.²</i>	<i>Million lb/in.²</i>	<i>Pct</i>			
TYPE A (5-1/8 by 16.5)								
A01	11	.46	5,900	2.18	100		Maj. at 13.7	Inv. f.j. with edge knot at 12.5 in 2nd lam.
A02	10	.46	5,930	2.15	100		Maj. at 14.2	
A03	12	.46	6,740	2.07	60		Maj. at 20.0	Maj. at f.j. at 11.6 in 2nd lam. and f.j. at 12.0 in 3rd lam. ⁵
A04	12	.45	5,080	2.06	100		Inv. at 11.4	Maj. at f.j. at 17.5 in 2nd lam. and knot at 17.0 in 3rd lam.
A05	12	.46	4,710	2.17	100		Maj. at 13.2	
A06	9	.46	8,500	2.02	55	Maj. at 10.6		Compression
A07	10	.46	4,980	2.03	95		Maj. at 10.3	
Ave.	11	.46	5,980	2.10	87			
COV	11.5	.8	22.0	3.2	--			
TYPE B (3-5/8 by 15.75)								
B01	13	.56	5,270	2.19	100		Maj. at 15.2	
B02	11	.56	6,540	1.89	85		10 pct at 17.6	
B03	13	.45	5,570	1.92	95		Maj. at 10.6	
B04	11	.56	4,500	1.98	100	Maj. at 14.7		Maj. at f.j. at 13.7 in 2nd lam. ⁶
B05	14	.44	4,190	2.06	100		Maj. at 13.0	Maj. at f.j. at 14.1 in 2nd lam.
B06	9	.48	8,870	2.22	100			Maj. at f.j. at 13.6 in 3rd lam. compression; Inv. knot at 13.1 in 2nd lam. ⁶
B07	12	.46	7,610	2.02	65	Maj. at 8.3	70 pct at 7.4	Maj. at f.j. at 8.2 in 2nd lam.
Ave.	12	.46	6,080	2.04	92			
COV	13.4	2.7	27.9	6.2	--			
TYPE C (2-1/8 by 15.0)								
C01	8	.44	5,370	1.85	90		Maj. at 9.9	Maj. at knot at 9.7 in 2nd lam.
C02	11	.45	5,250	1.91	70	Maj. at 13.6 and 15.9		
C03	11	.46	6,630	2.05	65	Maj. at 9.6		Maj. at f.j. at 10.2 in 2nd lam. and knot at 9.7 in 3rd lam.
C04	10	.46	4,440	2.03	100	⁶ Maj. at 14.4		Maj. at knot at 16.3 in 2nd lam.
C05	10	.46	6,480	1.94	75	Maj. at 12.8		
C06	10	.46	6,010	2.03	85	Maj. at 14.9 and 13.0		
C07	9	.46	5,190	2.10	80	Maj. at 16.0		
Ave.	10	.46	5,630	1.99	81			
COV	8.9	2.0	13.9	4.5	--			

¹Determined following test using a resistance-type meter with 1-1/2-inch-long needles. Data given are averages of readings taken for each lamination at point of failure.

²Based on weight and volume of beam at time of test. Weight was adjusted to oven-dry.

³Percent of the calculated maximum moment at the end joint.

⁴Locations are given in feet with reference to one end of the beam. Maj. = major cause of failure; Inv. = involved in failure; f.j. = finger joint.

⁵The space between the two end joints is less than 6 inches, and therefore does not meet PS-56-73.

⁶End joint in the maximum moment region of the tension lamination was not involved in the failure.