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BUCKLING COEFFICIENTS FOR FLAT, RECTANGULAR SANDWICH PANELS WITH CORRUGATED CORES UNDER EDGEWISE COMPRESSION

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ABSTRACT

This report presents curves of coefficients and formulas for use in calculating the buckling of flat panels of sandwich construction with corrugated cores under edgewise compressive loads. The formulas apply to sandwich panels having: (1) one facing of orthotropic material, the other facing of an isotropic material; (2) both facings of orthotropic material; or (3) both facings of isotropic material. Parameters are chosen so that facings may be of different thicknesses and so that facings can also be of different isotropic or orthotropic materials.

Equations are presented for various edge conditions. simply supported and clamped. Curves of buckling coefficients are presented for simply supported sandwich panels with both facings either isotropic or orthotropic.

BUCKLING COEFFICIENTS FOR FLAT, RECTANGULAR SANDWICH PANELS WITH CORRUGATED CORES UNDER EDGEWISE COMPRESSION [1]

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INTRODUCTION

The derivation of formulas for the buckling loads of rectangular sandwich panels subjected to edgewise compression is given in Forest Products Laboratory Report No. 1583-B.[3] These formulas are derived for the most gereral type of sandwich panel--panels with one facing of orthotropic material and the other of an isotropic material, and orthotropic cores--and for several combinations of simply supported and clamped edges. The cores are assumed to be of such a nature that stresses in them associated with strains in the plane of the panel may be neglected in comparison with the similar stresses in the facings, and that the elastic modulus normal to the facings is so great that the related strain may be neglected.

Compressive buckling curves for several kinds of sandwich panels were calculated and presented in U.S. Forest Service Research Note FPL-070.[4] These were panels with one facing of orthotropic material, the other facing of an isotropic material; both facings of orthotropic material; or both facings of isotropic material. Cores were either isotropic or orthotropic, with a finite shear modulus. Curves were presented for three different combinations of elastic properties of the orthotropic (glas-fabric laminate) facings and for four different combinations of panel edge support (simply supported or clamped).

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[1] This Paper is another progress report in the series (Military Handbook 23 Working Group, Item 58-2) prepared and distributed by the Forest Products Laboratory under U.S. Navy Bureau of Naval Weapons Order IPF 19-65-8005(WEPS) and U.S. Air Force Contract 33(615)65-5002. Results here reported are preliminary and may be revised as additional data become available.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3] Ericksen, W. S., and March, H. W. Effects of shear deformation in the core of a flat rectangular sandwich panel--Compressive buckling of sandwich panels having dissimilar fcings of unequal thickness. Forest Products Lab. Rpt. 1583-B. 48 pp., illus, revised 1958

[4] Kuenzi, Edward W., Norris, Charles B., and jenkinson, Paul M. Buckling coefficients for simply supported and clamped flat, rectangular sandwich panels under edgewise compression. U.S. Forest Serv. Res. Note FPL-070, 136 pp., illus,. 1964. Forest Products Lab., Madison Wis.

This report presents equations and buckling curves for sandwich panels having corrugated cores (a particular type of orthotropic core). Sketches of sandwich panels having corrugated cores are shown in figure 1. It is assumed that the core modulus of rigidity in the plane parallel to the corrugation flutes and perpendicular to the sandwich facings is infinitely stiff compared to the modulus of rigidity in the plane perpendicular to the corrugation flutes and perpendicular to the sandwich facings. Making use of this assumption, equations for buckling of sandwich panels having corrugated cores can be derived from the formulas given in Forest Products Laboratory Report 1583-E.[3] Buckling curves are presented for simply supported sandwich panels having isotropic or orthotropic facings, and having the core corrugations oriented parallel or perpendicular to the applied load.

FACING ELASTIC PROPERTIES

The various elastic properties of the facings can be combined into three convenient parameters for presentation of curves of buckling coefficients. These parameters are defined by the following:

$$\left.\begin{aligned} \alpha_i &= \sqrt{\frac{E_{bi}}{E_{ai}}} \\ \beta_i &= \alpha_i \mu_{abi} + 2\gamma_i \\ \gamma_i &= \frac{\lambda_i G_{abi}}{\sqrt{E_{ai} E_{bi}}} \end{aligned}\right\} \qquad (1)$$

where $\lambda_i = 1 - \mu_{abi}\mu_{bai}$; E_{ai} and E_{bi} and the moduli of elasticity of a facing in the a and b directions, respectively (see fig. 2); G_{abi} is the facing shear modulus associated with shear distortion in the plane of the facing (a-b plane); μ_{abi} is facing Poisson's ratio of contraction in the a direction to extension in the b direction due to a tensile stress in the b direction; μ_{bai} is facing Poisson's ratio of contraction in the b direction to extension in the a direction due to a tensile stress in the a direction; and i = 1,2, denotes facing 1 or facing 2.

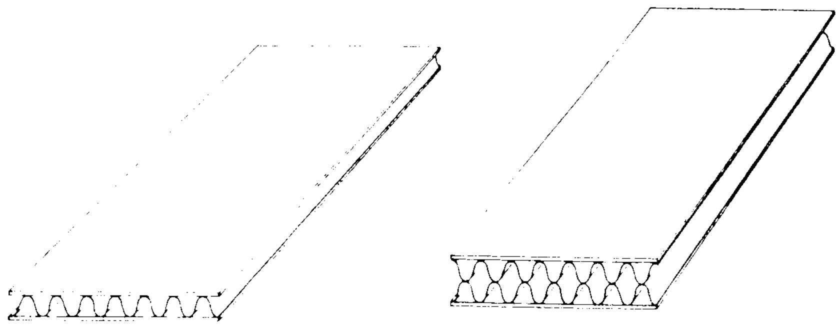

Figure 1.--Sandwich with corrugated cores: Single and double rows of corrugations.

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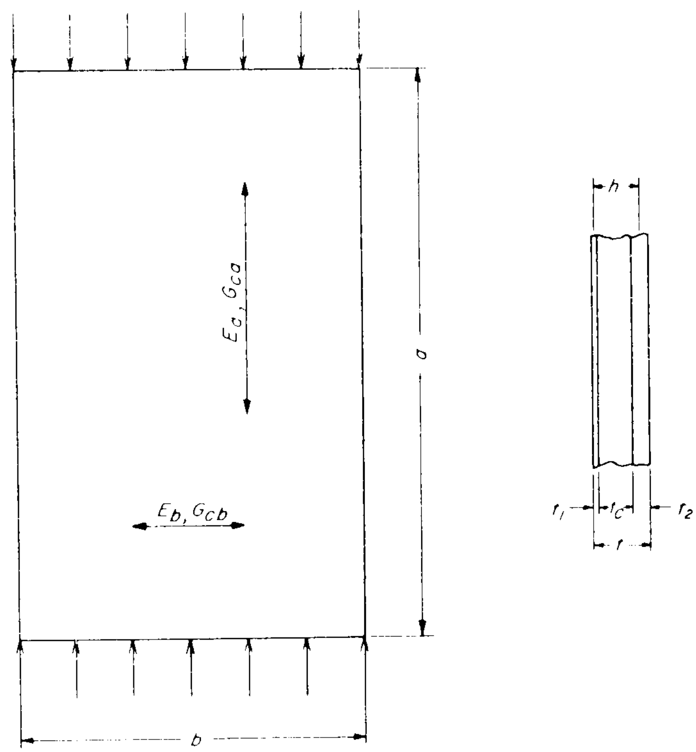

Figure 2.--Notation for dimensions and elastic properties of sandwich panel.

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For the computation of buckling coefficients presented in this report, facing 1 was taken to be isotropic and having a Poisson's ratio of $\mu_{bal} = \mu_{abl} = \mu_1 = 1/4$. For this facing, $E_{al} = E_{bl} = E_1$ and $G_{bal} = G_1 = \frac{E_1}{2(1+\mu_1)}$ and the parameters given by formulas (1) reduce to

$$\alpha_1 = 1; \beta_1 = 1; g_1 = 3/8$$

Facing 2 was taken to be orthotropic. A wide variety of materials could be selected to give a range of values of parameters a, ß, and γ but values of elastic properties of glass–fabric laminates were chosen. Data presented in Military Handbook–17[5] and combined to reasonable average values showed that elastic properties of polyester and epoxy laminates of glass fabrics 112, 116, 120, 128, 162, 164, 181, 182, 183, and 184 could be grouped to give parameter values of

$$\alpha_2 = 1, \beta_2 = 0.6, \text{ and } g_2 = 0.2$$

Although the values of these parameters originated in evaluation of glass–fabric laminates, they would apply to other orthotropic materials. Thus $\alpha_2 = 1$, $\beta_2 = 0.6$, and $\gamma_2 = 0.2$ would apply to any orthotropic material having $E_a = E_b = E$, $\mu_{ab} = 0.2$, $\mu_{ba} = 0.2$,[6] and $G_{ba} = 0.21E$.

FORMULAS

The following formulas apply to overall buckling of a sandwich panel. Sandwich with corrugated cores must also be designed to carry the axial load without overstressing facings or core, causing the facings to wrinkle into or away from the core, or causing local buckling of facing or core elements.

Either of two sets of buckling formulas for sandwich with corrugated core can be derived from the equations in Forest Products Laboratory Report 1583–B, depending on the orientation of the corrugated core.

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[5] U.S. Department of Defense. Plastics for flight vehicles––Part I, Reinforced plastics, Military Handbook 17. 1959.

[6] Assuming the relationship $E_a \mu_{ba} = E_b \mu_{ab}$.

Core Corrugations Parallel to Direction of Load

In this case the load is usually carried by both facings and core. Load is applied to two opposite edges of the panel, as shown in figure 2. The length of these edges is b and the length of the other two edges is a. The load is applied at the neutral axis of the panel so that the panel does not bend until the critical load is reached. It follows that the strains in the two facings and the core are equal and the critical stress in each facing is given by

$$F_{cl} = \frac{NA_1}{t_1 (A_1 + A_2 + A_c)} \tag{2}$$

and

$$F_{c2} = \frac{NA_2}{t_2 (A_1 + A_2 + A_c)} \tag{3}$$

and the corresponding core stress is given by

$$F_c = \frac{NA_c}{t_c (A_1 + A_2 + A_c)} \tag{4}$$

where N is the buckling load of the sandwich panel per unit width, t_1 is the thickness of the isotropic facing, and t_2 the thickncss of the orthotropic facing. The parameter A_i is given by

$$A_i = \frac{t_i}{\lambda_i} \sqrt{E_{ai} \; E_{bi}} \tag{5}$$

where i = 1,2 denotes facing 1 or facing 2. The parameter A_c is given by

$$A_c = \frac{t_c E_c}{\lambda_c} \tag{6}$$

where t_c is the thickness of the core, E_c is the effective modulus of elasticity of the core material in the direction of loading, and $\lambda_c = 1 - \mu_{ab}\mu_{ba}$ for the core.

Since the strains in facings and core must be equal, it follows that the buckling load of the sandwich per unit panel width, $\underline{N}$, is given by the formula

$$N = N_F \left(1 + \frac{A_c}{A1 + A2}\right) \tag{7}$$

where $\underline{N_F}$ is the critical load of the sandwich facings in pounds pes inch of edge.

The buckling load of the sandwich facings, per unit panel width, $\underline{N_F}$, is given by the formula[3]

$$N_F = K_M \frac{\pi^2}{b^2} D + K_1 \frac{p^2}{b^2} D_1 + K_2 \frac{\pi^2}{b^2} D_2 \tag{8}$$

where $\underline{b}$ is length of the loaded edge of the panel; $\underline{D}$ is stiffness of spaced lacings given by the formula

$$D = \frac{A_1 A_2}{A_1 + A_2} h^2 \tag{9}$$

where $\underline{h}$ is the distance between facing centroids, and $\underline{D_1}$ and $\underline{D_2}$ are stiffnesses of individual facings given by the formula

$$D_i = \frac{t_i^3 \sqrt{E_{ai} E_{bi}}}{12 \lambda_i} \tag{10}$$

where i = 1, 2 denotes facing 1 or facing 2, and the core is assumed to have negligible bending stiffness compared to sandwich stiffness but is able to carry edgewise toad.

$\underline{K_M}$, $\underline{K_1}$, and $\underline{K_2}$ may be calculated from the following formulas derived from expressions in reference ($\underline{3}$) after assuming $G_{ca} = \infty$:

$$K_M = \frac{\psi_1 K_2 + B_2 \cdot \frac{W}{c_4}}{\psi_2 + \psi_3(\alpha_2 c_1 + \gamma_2 c_2)\frac{W}{c_4}} \tag{11}$$

where $$\psi_1 = T + (1 - T)\frac{K_1}{K_2} \cdot \frac{B_2}{B_1} \tag{12}$$

$$\psi_2 = T^2 + 2T(1 - T)\frac{B_{12}}{B_1} + (1 - T)^2 \cdot \frac{B_2}{B_1} \tag{13}$$

$$\psi_3 = T + (1 - T)Q\frac{B_2}{B_1} \tag{14}$$

$$K_i = \alpha_i c_1 + 2\beta_i c_2 + \frac{c_3}{\alpha_i} \tag{15}$$

$$B_i = c_1 c_3 - \beta_i^2 c_2^2 + \gamma_i c_2 K_i \tag{16}$$

$$B_{12} = \left(\frac{\alpha_1^2 + \alpha_2^2}{2\alpha_1\alpha_2}\right) c_1 c_3 - \beta_1 \beta_2 c_2^2 + \frac{c_2}{2}(\gamma_1 K_2 + \gamma_2 K_1) \tag{17}$$

$$Q = \frac{\alpha_1 c_1 + \gamma_1 c_2}{\alpha_2 c_1 + \gamma_2 c_2} \tag{18}$$

The parameters of these formulas are given by the following expressions:

$$T \quad \frac{A_1}{A_1 + A_2} \tag{19}$$

$$W = \frac{A_1 A_2 \quad \pi^2 t_c}{\left(A_1 + A_2\right) \; b^2 G_{cb}} \tag{20}$$

where $\underline{G_{cb}}$ is the modulus of transverse rigidity of the core associated with the direction of the loaded edges (b) of the panel, as shown in figure 2, and $\underline{t_c}$ is the core thickness.

The values of c_1, c_2, c_3, and c_4 depend on the panel aspect ratio, $\frac{b}{a}$, the integral number of longitudinal half waves, n, into which the panel buckles, and the panel edge conditions. Values of n are chosen to produce minimum values of N_F.

For a panel with all edges simply supported:

$$c_1 = c_4 = \frac{a^2}{n^2 b^2}, \quad c_2 = 1, \text{ and } c_3 = \frac{n^2 b^2}{a^2}$$

For a panel with loaded edges simply supported and other edges clamped:

$$c_1 = \frac{16a^2}{3n^2 b^2}, \quad c_2 = \frac{4}{3}, \quad c_3 = \frac{n^2 b^2}{a^2}, \text{ and } c_4 = \frac{4a^2}{3n^2 b^2}$$

For a panel with loaded edges clamped and other edges simply supported:

$$\text{For } n = 1, \quad c_1 = c_4 = \frac{3a^2}{4b^2}, \quad c_2 = 1, \quad c_3 = 4\frac{b^2}{a^2}$$

$$\text{For } n \geq 2, \quad c_1 = c_4 = \frac{a^2}{(n^2 + 1)b^2}, \quad c_2 = 1, \quad c_3 = \frac{n^4 + 6n^2 + 1}{n^2 + 1} \cdot \frac{b^2}{a^2}$$

For a panel with all edges clamped:

$$\text{For } n = 1, \quad c_1 = 4c_4 = 4\frac{a^2}{b^2}, \quad c_2 = \frac{4}{3}, \quad c_3 = 4\frac{b^2}{a^2}$$

$$\text{For } n \geq 2, \quad c_1 = 4c_4 = \frac{16a^2}{3(n^2 + 1)b^2}, \quad c_2 = \frac{4}{3}, \quad c_3 = \frac{n^4 + 6n^2 + 1}{n^2 + 1} \cdot \frac{b^2}{a^2}$$

Core Corrugations Perpendicular
To Direction of Load

In this case all the edgewise load is carried by the facings because it is assumed that $E_c = 0$, hence $A_c = 0$. The critical stresses in the facings are again given by equations (2) and (3) and the buckling load of the sandwich per unit panel width, N, is given by equation (8) since $N = N_F$.

The buckling coefficient K_M may be calculated from the formula:

$$K_M = \frac{\psi_1 K_2 + B_2 V}{\psi_2 + \psi_3 \left(c_2 \gamma_2 + \frac{c_3}{\alpha_2}\right) V} \tag{21}$$

where ψ_1, ψ_2, ψ_3, K_i, B_i, B_{12}, and T are as defined previously.

$$Q = \frac{c_2 \gamma_1 + \frac{c_3}{\alpha_1}}{c_2 \gamma_2 + \frac{c_3}{\alpha_2}} \tag{22}$$

and the parameter V is given by

$$V = \frac{A_1 A_2}{\left(A_1 + A_2\right)} \frac{\pi^2 t_c}{b^2 G_{ca}} \tag{23}$$

where G_{ca} is the modulus of transverse rigidity associated with the direction of the unloaded edges (a) of the panel, as shown in figure 2.

The values of c_1, c_2, c_3, and c_4 are as previously defined.

DISCUSSION OF CURVES OF BUCKLING COEFFICIENTS

Buckling coefficients $\underline{K_M}$ were computed for simply supported sandwich panels and are presented in figures 3 to 6. Figures 3 and 4 are for sandwich panels with the core corrugation flutes parallel to the unloaded edges of the panels, and figures 5 and 6 apply to sandwich having the core corrugation flutes parallel to the loaded edges of the panel.

The figures contain a family of curves consisting of a plot of $\underline{K_M}$ against $\underline{a/b}$ for various values of $\underline{V}$ or $\underline{W}$. Each cusped curve is made up of portions of the curve for the $\underline{n}$ which gives the least value of $\underline{K_M}$. The parameter $\underline{a/b}$ is used in the left half of the curve sheets, and the parameter $\underline{b/a}$ in the right half. Thus values of $\underline{K_M}$ for values of $\underline{a/b}$ from zero to infinity may be read.

Limits

For core corrugation flutes parallel to the loaded edges, certain limits may be determined. When $\underline{a/b}$ is zero, minimum values are given by $K_M = \frac{1}{V} + \gamma$. For other values of $\underline{a/b}$, as the value of $\underline{V}$ increases, the value of $\underline{K_M}$ decreases and the minimum points on the curve move to the left. There is a value of $\underline{V}$ for which the first minimum point of the curve occurs at $\underline{a/b}$ equal to zero, the $\underline{K_M}$ intercrpt. This minimum point is common to the curves associated with all numbers of half waves. Of these curves, the curve for an infinite number of half waves yields the least critical value and is a horizontal straight line. These straight lines are shown on the curve sheets of figures 5 and 6. If $\underline{V}$ is given a value equal to or greater than that associated with these straight lines, the critical value of $\underline{K_M}$ is $\frac{1}{V} + \gamma$. For simply supported panels the first straight line occurs for $V = \frac{1}{1 + \beta - \gamma}$ for which $K_M = \frac{1}{V} + \gamma = 1 + \beta$.

The parameter $\underline{T}$ (formula (17)) was devised[3] as a means of convenience in handling the analysis and presentation of results for sandwich with dissimilar facings. The role of $\underline{T}$ and its range of values can be understood most easily by examining its place in the parameters of the KM expressions. $\underline{T}$ appears in the equations for ψ_1, ψ_2, and ψ_3 (equations (12, 13, and 14)). If T = 0 is substituted into these expressions, they become

$$\psi_1 = \frac{K_1 B_2}{K_2 B_1};\ \psi_2 = \frac{B_2}{B_1};\ \psi_3 = Q\frac{B_2}{B_1}$$

and substitution of these expressions into formula (11) for K_M results in

$$K_M = \frac{K_1 + B_1 \cdot \frac{W}{c_4}}{1 + \left(\alpha_1 c_1 + \gamma_1 c_2\right) \frac{W}{c_4}} \tag{24}$$

Substitution of these same three expressions into formula (21) for K_M results in

$$K_M = \frac{K_1 + B_1 V}{1 + \left(c_2 \gamma_1 + \frac{c_3}{\alpha_1}\right) V} \tag{25}$$

Formulas (24) and (25) are dependent only upon the properties of facing 1 as indicated by the appearance of only 1 as subscripts on terms involving facing properties. Hence, T = 0 establishes a limit wherein both facings are of type 1 material having the same values of α_1, β_1, and γ_1 but not necessarily having the same elastic moduli or thickness. In this report, type 1 material is defined as being isotropic.

If T = 1 is substituted into the equations for ψ_1, ψ_2, and ψ_3 (equations (12, 13, and 14)), they become

$$\psi_1 = 1; \; \psi_2 = 1; \; \psi_3 = 1$$

and the resulting expression for equation (11) for K_M becomes

$$K_M = \frac{K_2 + B_2 \cdot \frac{W}{c_4}}{1 + \left(\alpha_2 c_1 + \gamma_2 c_2\right) \frac{W}{c_4}} \tag{26}$$

and the formula (21) becomes

$$K_M = \frac{K_2 + B_2 V}{1 + \left(c_2 \gamma_2 + \frac{c_3}{\alpha_2}\right) V} \tag{27}$$

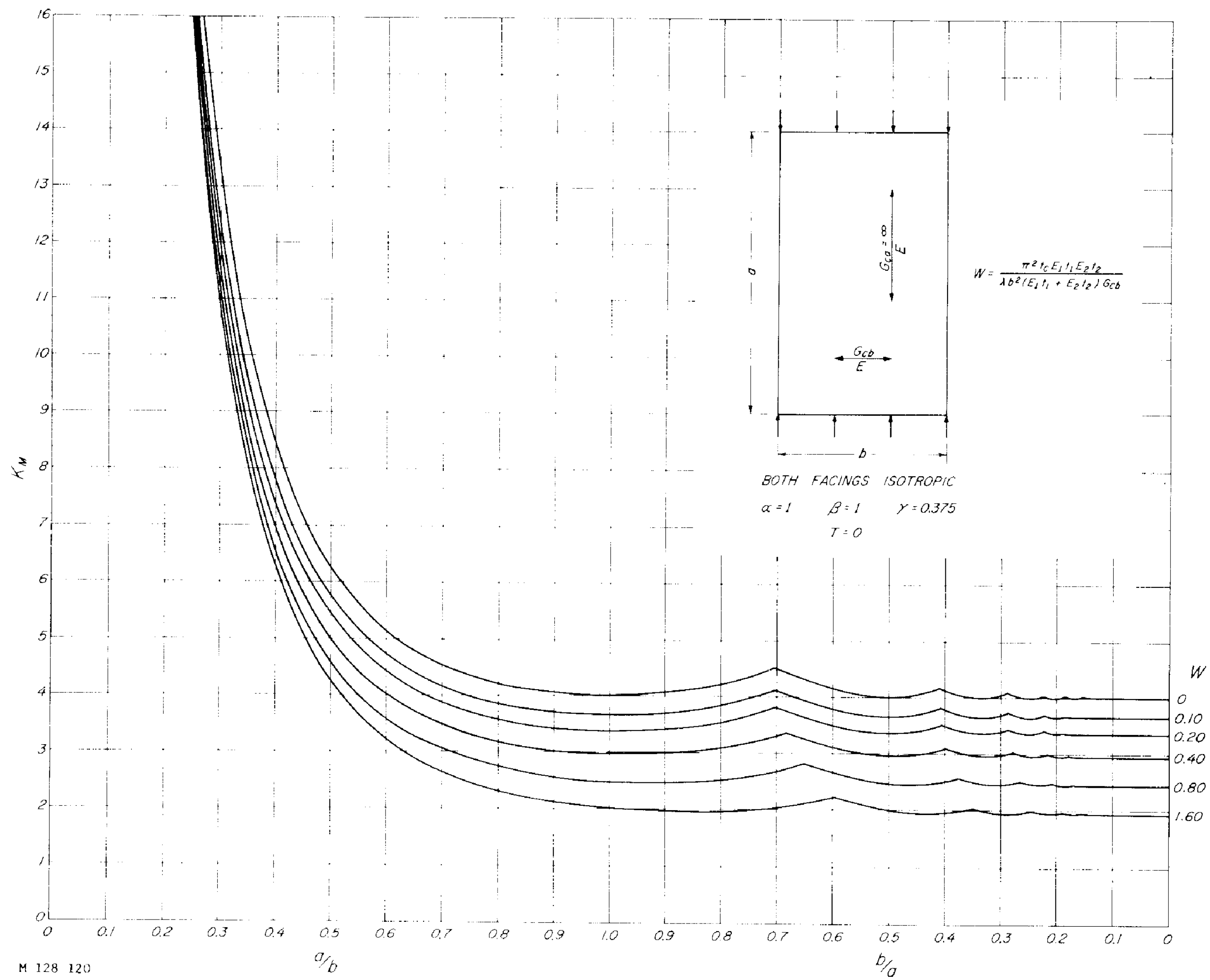

Figure 3.--Edgewise compression buckling coefficients for simply supported sandwich panels having an isotropic facing and a corrugated core; direction of core corrugation flutes parallel to unloaded edges.

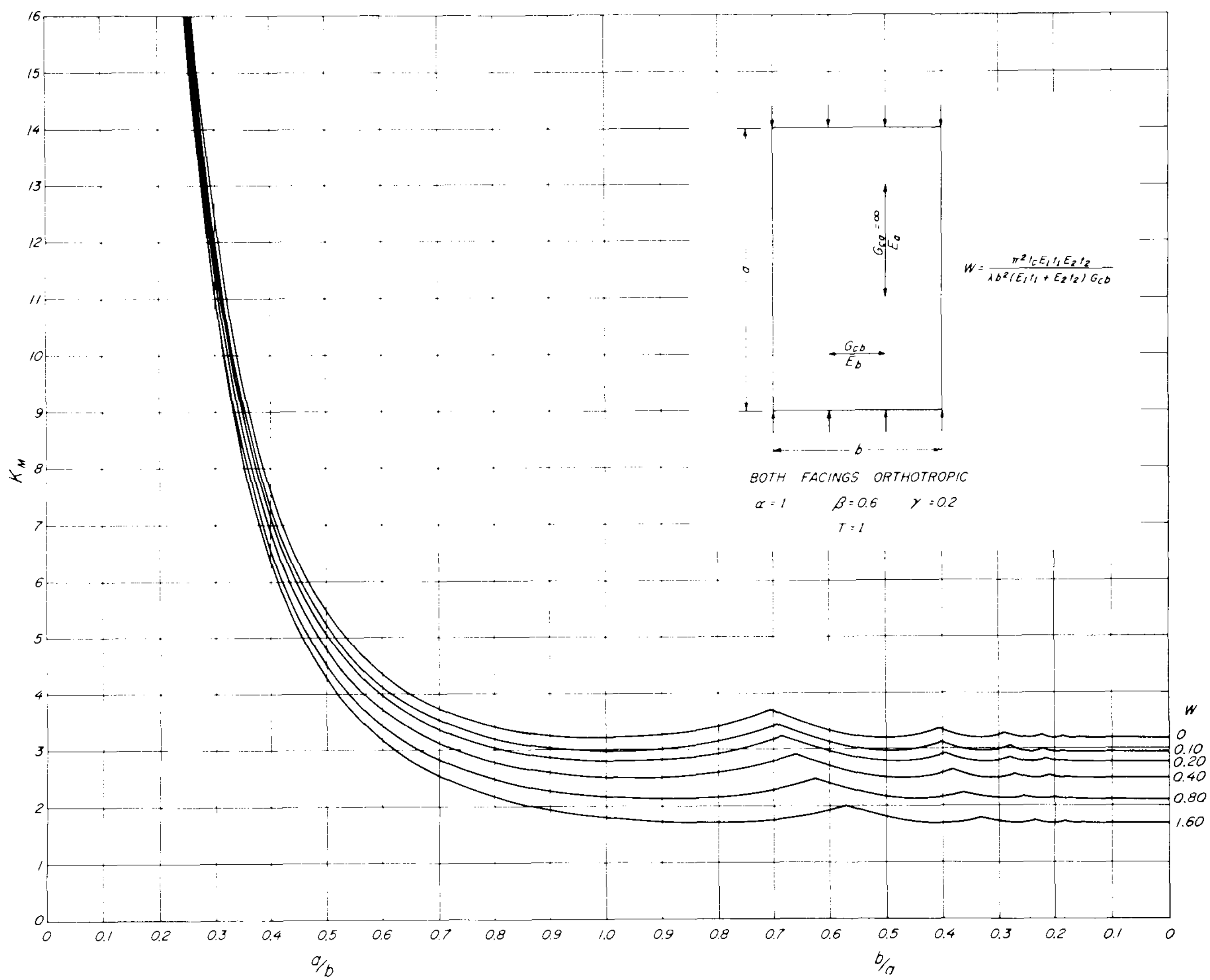

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Figure 4.--Edgewise compresssion buckling coefficients for simply supported sandwich panels having an orthotropic facing and a corrugated core; direction of core corrugation flutes parallel to unloaded edges.

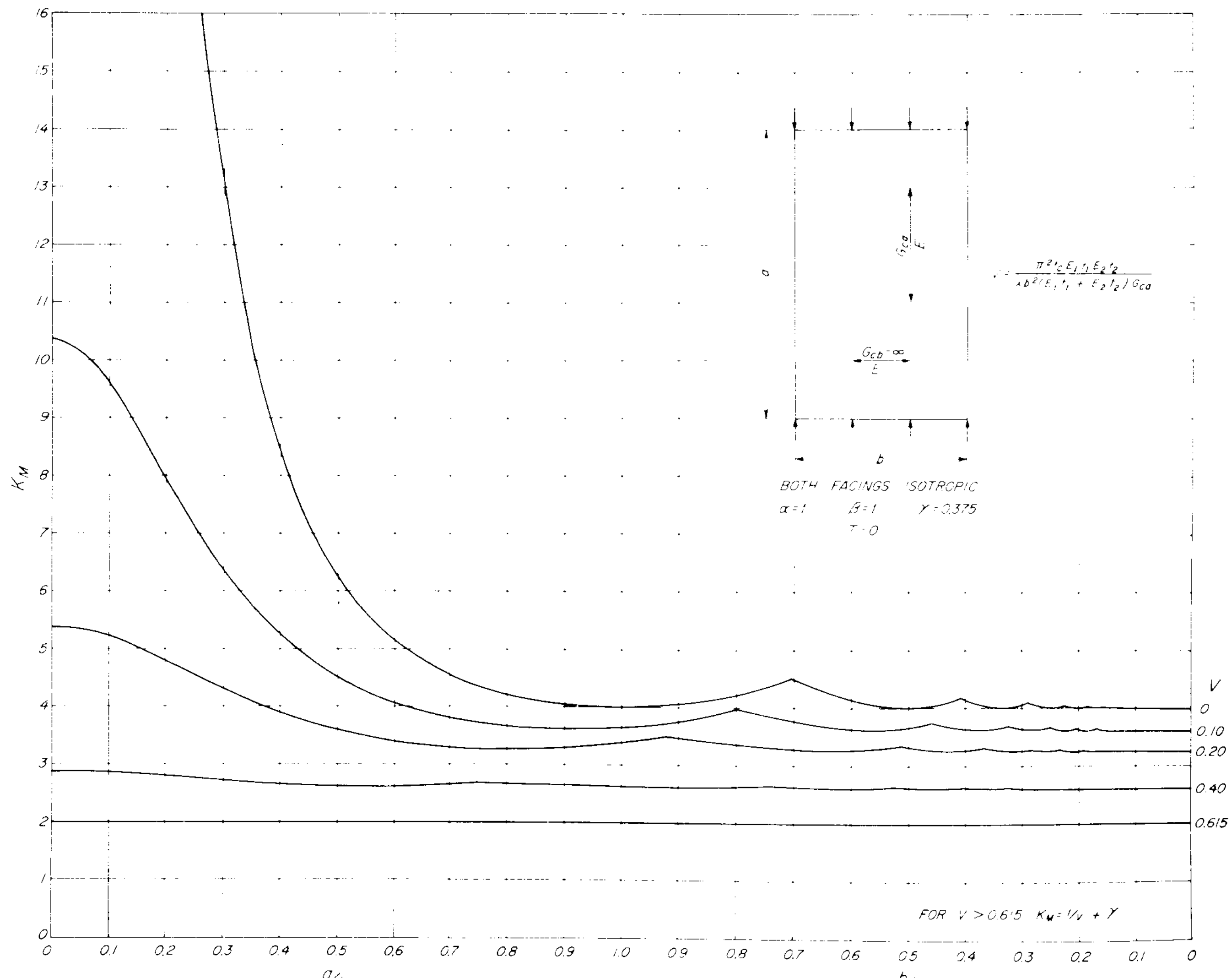

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Figure 5.--Edgewise compression buckling coefficients for simply supported sandwich panels having an isotropic facing and a corrugated core; direction of core corrugation flutes parallel to loaded edges.

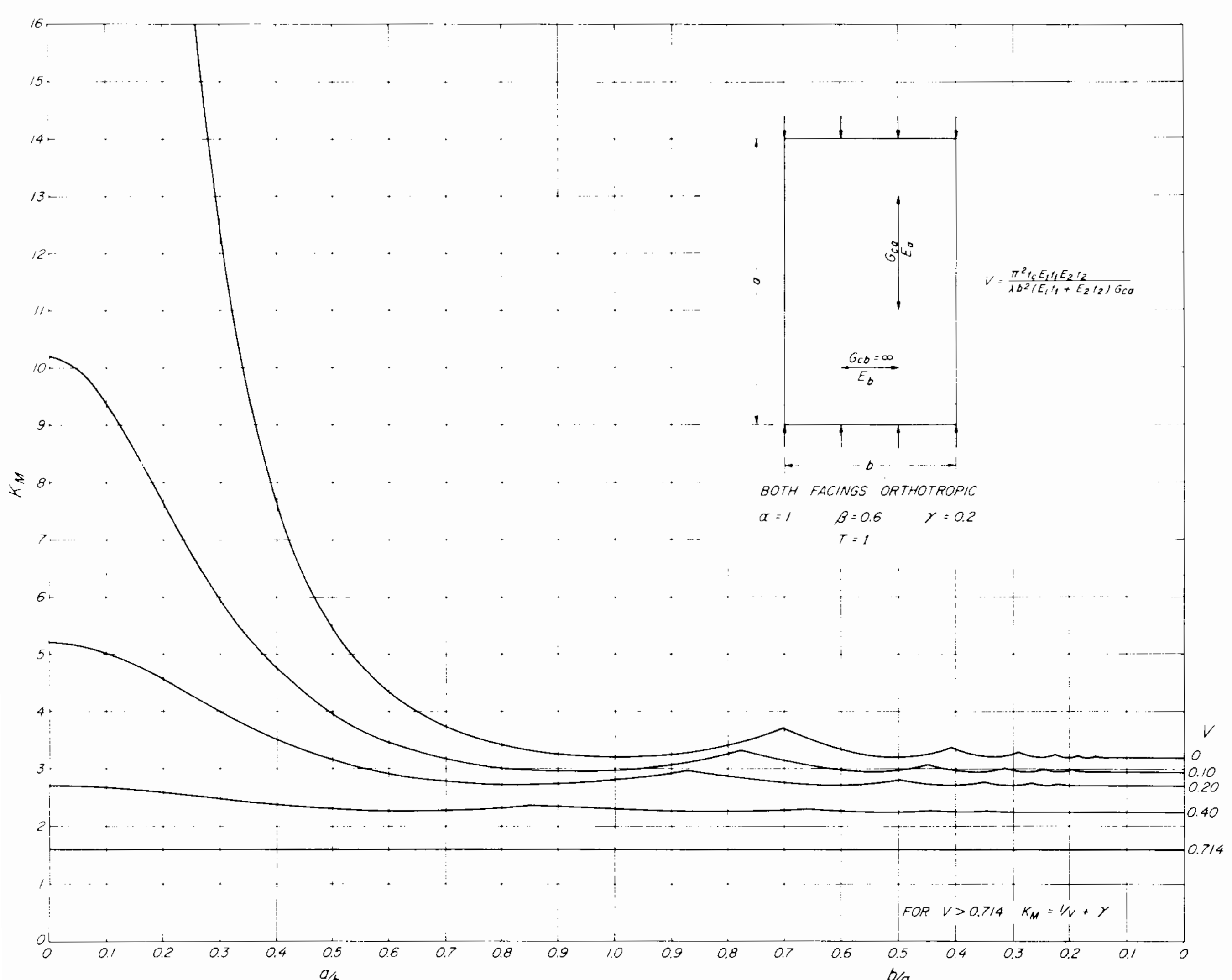

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Figure 6.--Edgewise compression buckling coefficients for simply supported sandwich panels having an orthotropic facint and a corrugated core: direction of core corrtgation flutes parallel to loaded edges.

These expressions are dependent only upon the properties of facing 2. Hence, T = 1 establishes a limit wherein both facings are of type 2 material having the same values of $\underline{\alpha_2}$, $\underline{\beta_2}$, and $\underline{\gamma_2}$ but not necessarily having the same elastic moduli or thickness. In this report, type 2 material is defined as being orthotropic.

The buckling coefficients given in figures 4 and 6 for panels having orthotropic facings were calculated for $\alpha_2 = 1$, $\beta_2 = 0.6$, and $\gamma_2 = 0.2$.

For sandwich panels having one facing isotropic and one facing orthotropic, the value of the parameter $\underline{T}$ lies between 0 and 1. Buckling coefficients for such panels may be determined by linear interpolation between values of $\underline{K_M}$ for T = 0 and T = 1 presented in figures 3 through 6.

It should not be overlooked that for $T \neq 0$ or $T \neq 1$ the facings of the sandwich are dissimilar. For example, it is not sufficient to place $A_1 = A_2$ in the definition of T and suppose that T = 1/2 represents sandwich with similar facings because setting $A_1 = A_2$ does not result in $K_1 = K_2$ or $B_1 = B_2$ etc., as is necessary for sandwich with similar facings.

The foregoing has discussed analysis and curves presented for the buckling coefficient $\underline{K_M}$ because the primary portion of the buckling load $\underline{N_F}$ is given by the term of formula (8) containing $\underline{K_M}$. The other two terms of formula (8) involving stiffness of individual facings, $\underline{D_1}$ and D_2 have buckling coefficients $\underline{K_1}$ and $\underline{K_2}$. The $\underline{K_1}$ and $\underline{K_2}$ coefficients can be obtained from the $\underline{K_M}$ curves for T = 0 and T = 1, respectively, for W = 0 or V = 0. This can be seen by examining formulas (24) and (26) for W = 0 and formulas (25) and (27) for V = 0. The effects of adding individual facing stiffness will usually be small unless the panel is very short. For convenience and greatere accuracy in cinsidering buckling of short panels, logarithmic plots of $\underline{K_1}$ and $\underline{K_2}$ as functions of $\underline{a/b}$ are given in figure 7.

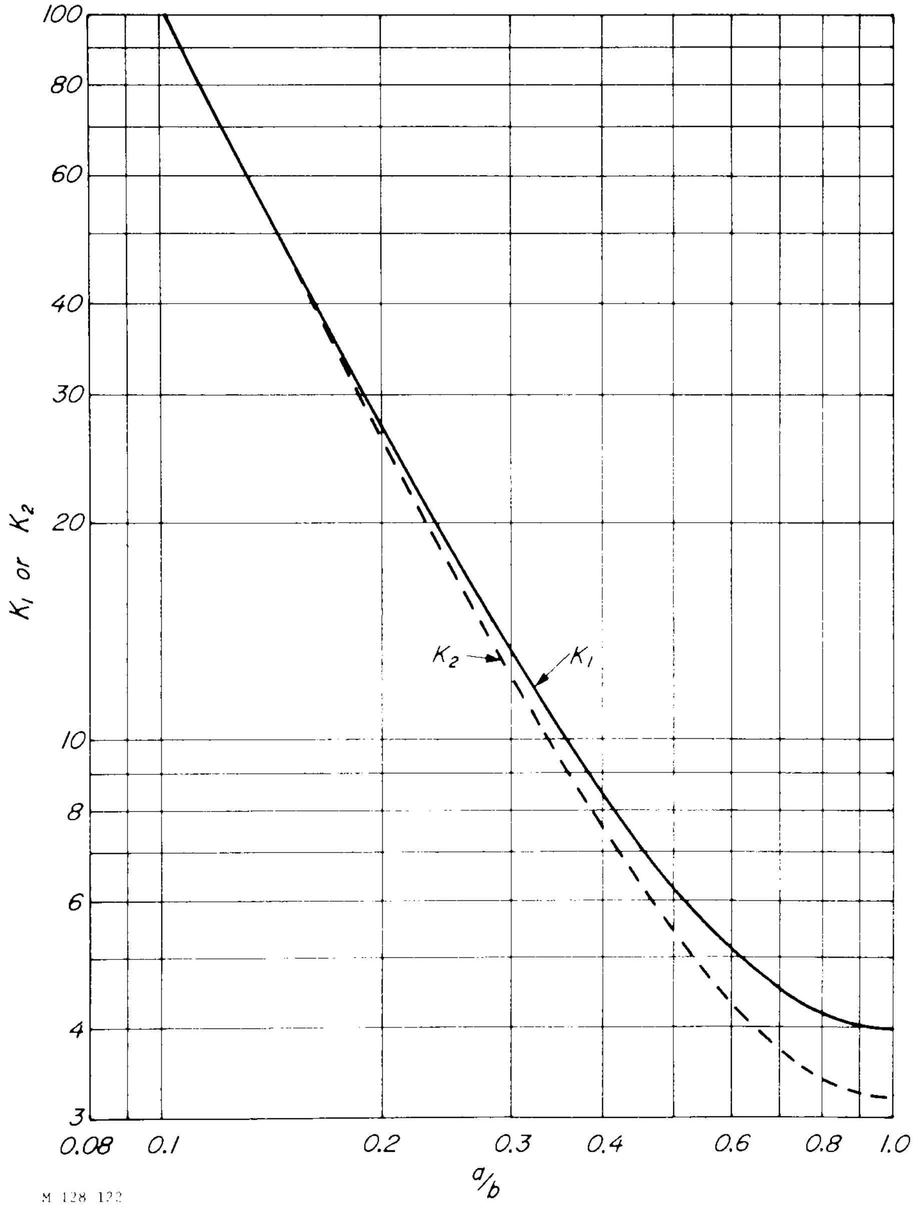

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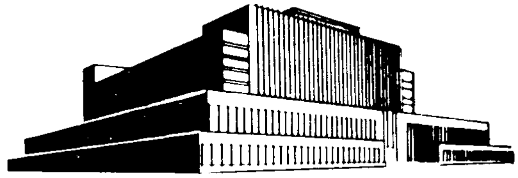
Figure 7.--Values of K_1 and K_2, (n = 1), for sandwich panels with ali edges simply supported, $\alpha_1 = 1$, $\beta_1 = 1$, $\gamma_1 = 0.375$, $\alpha_2 = 1$, $\beta_2 = 0.6$, $\gamma_2 = 0.2$.



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