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CLASSICAL BUCKLING OF CYLINDERS OF SANDWICH CONSTRUCTION IN AXIAL COMPRESSION--ORTHOTROPIC CORES

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CLASSICAL BUCKLING OF CYLINDERS OF SANDWICH CONSTRUCTION IN AXIAL COMPRESSION--ORTHOTROPIC CORES[1]

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Abstract

Classical small deflection theory is used to derive buckling coefficients for cylinders of sandwich construction under axial compression. The core is assumed to be orthotropic, but the results do not differ much from isotropic core unless the core shear modulus is much smaller circumferentially than axially. The buckling coefficients for isotropic core and orthotropic core stiffened circumferentially are the same and are presented in a particularly simple formula:

$$K = 1 - V_x, \quad V_x \leq 1/2$$

$$K = \frac{1}{4V_x}, \quad V_x \geq 1/2$$

Introduction

This report assumes a sandwich construction of thin isotropic facings bonded to an orthotropic anti-plane core. It was decided not to use a differential equation method for the problem since energy methods, if well understood, permit the early insertion of simplifying assumptions that greatly shorten the solution. Further, the energy method will yield the "exact" answer if

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[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

the form of the buckled surface that satisfies the differential equations and boundary conditions is known in advance, as it is here. Of course, the word "exact" must here be understood in a restricted sense since this is a small deflection theory and even the "exact" solution represents an upper limit to the buckling load.

In a pioneering application of large deflection theory to a thin homogeneous shell, Von Karman and Tsien (6)[3] used an energy method which, for the limiting case of small displacements, exactly checked the result given by Timoshenko (5). This indicates that the assumptions in Von Karman and Tsien's energy method are just sufficient to eliminate from the final result all the terms that Timoshenko discards as small in simplifying his final result. The two assumptions are these:

(1) That the equilibrium equations of a curved membrane can be approximated by those of a flat membrane, permitting the use of an Airy stress function in computing the energy associated with extensional strains in the shell, and

(2) That the changes of curvature and unit twist of the shell can be approximated by the second derivatives of the radial displacement, omitting the terms containing the circumferential displacement. In this case the initial curvature of the shell is accounted for solely through the use of curvilinear coordinates; the expressions look exactly like those for flat plates. Donnell (1) shows that the neglected terms are of the order $\frac{1}{n^2}$ times the terms retained, where n is the number of waves in the buckled surface in the circumferential direction.

The assumption of anti-plane stress in the core is incorporated into the energy method here employed by assuming core displacements in the manner used by Williams, Leggett, and Hopkins in their analysis of flat sandwich panels (7). This is the so-called "tilting method" by which the flexural energy of flat sandwich panels was developed in Forest Products Laboratory Report No. 1583-B (2). In keeping with the assumptions stated above, the results of that report are used here.

The method of Von Karman and Tsien has been extended to sandwich construction by March and Kuenzi (3). To simplify the analysis, however, March and Kuenzi assumed the radial displacement in a form that does not reduce to the form required by classical theory when the displacements are small.

It is shown by Von Karman and Tsien, and again by March and Kuenzi for sandwich construction, that under the simplifying assumptions stated the mean circumferential stress in the shell is zero. This result is independent of whether the deflections are large or small, and so no provision is made for mean circumferential stress in this analysis.

[3] Underlined numbers in parentheses refer to the references.

Notation

A_i, $i = 1, 5$	See equations (20)
t_c	Core thickness
C	Wave amplitude of buckled surface
D	Flexural rigidity of sandwich, see (22)
E	Facing modulus of elasticity
e_x, e_y	Extensional strains of facings
Φ	Airy stress function
G_{xz}, G_{yz}	Shear moduli of core
h	$t_c = \frac{t_1 + t_2}{2}$
k	See (11)
K	Dimensionless buckling coefficient, see (29)
a	Length of buckled cylinder
m	Number of half-waves in axial direction
n	Number of waves in circumferential direction
f	Mean axial compressive stress in facings
r	Radius of cylinder
S_x, S_y	Core shear parameters, see (22)
t_1, t_2	F acing thicknesses
U	Total potential energy
u, v, w	Displacements in x, y, z directions

Notation

V_X	Core shear parameter, see (30)
W_1, W_2, W_3	Energy terms
x, y, z	Coordinates, see figure 1
γ_{xy}	Shear strain of facing
σ_x, σ_y, σ_{xy}	Facing stresses
Δ^2	
λ	$\frac{m \pi r}{a}$
ξ	$\left(\frac{na}{m \pi r}\right)^2$
η	$\frac{a^2 \sqrt{1 - \nu^2}\ (t_1 + t_2)}{2m^2 \pi^2 rh \sqrt{t_1 t_2}}$
ϕ	$\frac{t_c t_1 t_2}{t_1 = t_2}$
θ	$\frac{G_{xz}}{G_{yz}}$
ν	Poisson's ratio of facings
X_x, X_y, X_{xy}	Curvatures and unit twist

Theretical Analysis

Choice of Axes

The choice of axes is shown in figure 1, the y axis being measured along the circumference. Thus x ranges from 0 to a and y ranges from 0 to $2\pi r$. Note the z axis is directed radially inward. Corresponding to x, y, and z are displacements u, v, and w. The dimension r is measured to the middle surface of the shell, as shown in figure 2. The distance q locates the neutral axis in circumferential bending.

Assumed Form of Radial Displacement

Classical theory of thin homogeneous shells requires that the buckled form of equilibrium be

$$w = Cw_1$$

$$w_1 = \sin\frac{ny}{r}\ \sin\frac{m\pi x}{a} \qquad (1)$$

where n is the number of waves in the circumferential direction and m is the number of half-waves in the axial direction. Introducing

$$\lambda = \frac{m\pi r}{a} \qquad (2)$$

this can be written

$$w_1 = \sin\frac{ny}{r}\ \sin\frac{\lambda x}{r} \qquad (3)$$

Strictly speaking, this radial displacement is in addition to a mean radial expansion, $\overline{w}$. Since this is an energy method and only the derivatives of w appear in the simplified energy expressions, however, the mean radial expansion can be ignored.

Energy of Extensional Strains, W_1

The total state of strain of the shell is written as the sum of two parts: an extensional strain that is uniform across the thickness of the shell (i. e., same in each facing) and flexural strains. The total strain energy of the shell is approximated by finding the energy associated with each part and simply adding. Since this is to be a small deflection theory, the linear strain–displacement relations for the extensional strains are as follows:

$$e_x = \frac{\partial u}{\partial x}$$

$$e_y = \frac{\partial v}{\partial y} - \frac{w}{r} \qquad (4)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

The core stresses associated with these strains are essentially zero and are ignored. This yields the so–called anti–plane core. Since these strains are the same in each facing, the shell can be considered here as a membrane whose thickness is the sum of the facing thicknesses. For isotropic facing material the stress–strain relationships are

$$e_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$e_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \qquad (5)$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$

These stresses satisfy equations of equilibrium that can be approximated by the equilibrium equations of a flat membrane:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \qquad (6)$$

This pair of equations is exactly satisfied if the stresses are derived from an Airy stress function:

$$\sigma_x = \frac{\partial^2 \Phi}{\partial y^2} \quad , \quad \sigma_y = \frac{\partial^2 \Phi}{\partial x^2} \quad , \quad \tau_{xy} = -\frac{\partial^2 \Phi}{\partial x \partial y} \qquad (7)$$

Unknowns $\underline{u}$ and $\underline{v}$ can be eliminated from equations (4) as follows:

$$\frac{\partial^2 e_x}{\partial y^2} + \frac{\partial^2 e_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = -\frac{1}{r} \frac{\partial^2 w}{\partial x^2} \qquad (8)$$

Equation (8) is called the compatibility condition for strains and must be satisfied if the displacements are single valued functions. Substituting from (5) into (8) the compatibility condition in terms of stress is:

$$\frac{1}{E}\left[\frac{\partial^2 \sigma_x}{\partial y^2} - \nu \frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial x^2} - \nu \frac{\partial^2 \sigma_y}{\partial y^2} - 2(1+\nu)\frac{\partial^2 \tau_{xy}}{\partial x \partial y}\right] = -\frac{1}{r} \frac{\partial^2 w}{\partial x^2}$$

and, using (7), this becomes

$$\nabla^2 \nabla^2 \Phi = -\frac{E}{r} \frac{\partial^2 w}{\partial x^2} \qquad (9)$$

Using the assumed displacement (3) this becomes

$$\nabla^2 \nabla^2 \Phi = \frac{EC}{r} \left(\frac{\lambda}{r}\right)^2 \sin\frac{ny}{r} \sin\frac{\lambda x}{r} \qquad (10)$$

As stated in the introduction, the work of previous investigators (6,3) shows that the mean radial expansion $\overline{w}$ is just sufficient to insure that the mean circumferential stress is zero. Hence, in the solution of (10) the term that produces a uniform stress in the y direction is discarded. Then the solution of (10) is

$$\Phi = kC \sin\frac{ny}{r} \sin\frac{\lambda x}{r} - \frac{fy^2}{2} + C_1 \frac{x^2}{2}$$

where C_1--mean circumferential stress equals 0

$$k = \frac{rE\lambda^2}{(\lambda^2 + n^2)^2} \qquad (11)$$

f--mean compressive stress in axial direction.

From (11), (7), and (1)

$$\sigma_x = - f + k C \frac{\partial^2 w_1}{\partial y^2}, \quad \sigma_y = k C \frac{\partial^2 w_1}{\partial x^2}, \quad \tau_{xy} = - k C \frac{\partial^2 w_1}{\partial x \partial y} \tag{12}$$

The extensional elastic energy corresponding to these stresses is

$$W_1 = \frac{t_1 + t_2}{2E} \int_0^{2\pi r} \int_0^{a} \left\{ (\sigma_x + \sigma_y)^2 - 2(1 + \nu)(\sigma_x \sigma_y - \tau_{xy}^2) \right\} dx\, dy \tag{13}$$

Using (12), (13) becomes

$$W_1 = \frac{t_1 + t_2}{2E} \int_0^{2\pi r} \int_0^{a} \left\{ (- f + k C \nabla^2 w_1)^2 - 2(1 + \nu) \left[(- f + k C \frac{\partial^2 w_1}{\partial y^2}) k C \frac{\partial^2 w_1}{\partial x^2} - k^2 C^2 (\frac{\partial^2 w_1}{\partial x \partial y})^2 \right] \right\} dx\, dy \tag{14}$$

The energy condition of equilibrium is that the total potential energy U of the system be a minimum. Hence, the displacement parameter C must be ad– justed so as to make U a minimum:

$$\frac{\partial U}{\partial C} = 0 \tag{15}$$

Since W_1 is a term in U then

$$\frac{\partial W_1}{\partial C} = \frac{t_1 + t_2}{E} \int_0^{2\pi r} \int_0^{a} \left\{ (- f + k C \nabla^2 w_1) k \nabla^2 w_1 - (1 + \nu) \left[(- f + k C \frac{\partial^2 w_1}{\partial y^2}) k \frac{\partial^2 w_1}{\partial x^2} + k^2 C \frac{\partial^2 w_1}{\partial x^2} \frac{\partial^2 w_1}{\partial y^2} - 2 k^2 C (\frac{\partial^2 w_1}{\partial x \partial y})^2 \right] \right\} dx\, dy = \frac{k^2 C (t_1 + t_2)}{E r^4} (\lambda^2 + n^2)^2 \frac{a \pi r}{2} \tag{16}$$

where the fact has been used that:

$$\int_0^{2\pi r} \int_0^{a} \sin \frac{ny}{r} \sin \frac{m\pi x}{a} \, dx \, dy = 0$$

and

$$\int_0^{2\pi r} \int_0^{a} \left\{ \begin{matrix} \sin^2 \frac{ny}{r} \\ \text{or} \\ \cos^2 \frac{ny}{r} \end{matrix} \right\} \left\{ \begin{matrix} \sin^2 \frac{m\pi x}{a} \\ \text{or} \\ \cos^2 \frac{m\pi x}{a} \end{matrix} \right\} dx \, dy = \frac{a\pi r}{2} \tag{17}$$

<u>Energy of Flexural Strains, W_2</u>

Following Donnell (<u>1</u>) the following simplified expressions are used for the changes in curvature and unit twist:

$$X_x = \frac{\partial^2 w}{\partial x^2}, \quad X_{xy} = \frac{\partial^2 w}{\partial x \partial y}, \quad X_y = \frac{\partial^2 w}{\partial y^2} \tag{18}$$

This neglects terms containing $\frac{V}{r}$. Donnell shows that the error is of the order $\frac{1}{n^2}$ times the terms retained where <u>n</u> equals number of waves in circumferential direction. Since expressions (18) are exactly those used in calculating the flexural energy of a flat plate in Forest Products Laboratory Report No. 1583-B (<u>2</u>), the results of that report are used. With these,

(19)

where

$$A_1 = \frac{E}{1-\nu^2} \int_0^{2\pi r} \int_0^{a} \left\{ \left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 + \frac{1-\nu}{2} \left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 \right\} dx\, dy$$

$$A_2 = \frac{E}{1-\nu^2} \int_0^{2\pi r} \int_0^{a} \left\{ \nu \frac{\partial^2 w_1}{\partial x^2} \frac{\partial^2 w_1}{\partial y^2} + \frac{1-\nu}{2} \left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 \right\} dx\, dy$$

$$A_3 = \frac{E}{1-\nu^2} \int_0^{2\pi r} \int_0^{a} \left\{ \left(\frac{\partial^2 w_1}{\partial y^2}\right)^2 + \frac{1-\nu}{2} \left(\frac{\partial^2 w_1}{\partial x \partial y}\right)^2 \right\} dx\, dy \qquad (20)$$

$$A_4 = G_{xz} \int_0^{2\pi r} \int_0^{a} \left(\frac{\partial w_1}{\partial x}\right)^2 dx\, dy$$

$$A_5 = G_{yz} \int_0^{2\pi r} \int_0^{a} \left(\frac{\partial w_1}{\partial y}\right)^2 dx\, dy$$

Report 1583-B (2) is based on the tilting method of Williams, Leggett, and Hopkins (7). In using (A31) through (A36) of that report the energy associated with the bending of the facings about their own middle surfaces is ignored.

From (3), (19), and (20),

$$W_2 = \frac{DC^2}{2} \cdot \frac{a\pi r}{2} \cdot \frac{1}{r^4}$$

$$\cdot \frac{(\lambda^2 + n^2)^2 \left[1 + \frac{1-\nu}{2} (n^2 + \lambda^2 \theta) S_x \frac{h}{r}\right]}{1 + (\lambda^2 + \frac{1-\nu}{2} n^2) S_x \frac{h}{r} + (n^2 + \frac{1-\nu}{2} \lambda^2) \theta S_x \frac{h}{r} + \frac{1-\nu}{2} (\lambda^2 + n^2)^2 \theta S_x^2 \frac{h^2}{r^2}} \qquad (21)$$

where

$$D = \frac{E t_1 t_2 h^2}{(1-\nu^2)(t_1 + t_2)}$$

$$S_x = \frac{E t_c t_1 t_2}{(1-\nu^2)(t_1 + t_2)\, h\, r\, G_{xz}} \qquad (22)$$

$$S_y = \theta S_x \;, \quad \theta = \frac{G_{xz}}{G_{yz}}$$

Work Done by the Load, W_3

Let $$W_3 = W_3' + W_3'' \qquad (23)$$

where W_3' is work done before buckling

W_3'' is work done during buckling

The assumed displacement (1) carries the shell from the prebuckled state to the postbuckled state. The work done during this process is independent of C, since the displacements have been assumed to be small. This can be seen as follows:

$$W_3'' = - f\,(t_1 + t_2) \int_0^{2\pi r} \int_0^{a} \frac{\partial u}{\partial x}\, dx\, dy$$

$$= - f\,(t_1 + t_2) \int_0^{2\pi r} \int_0^{a} \frac{1}{E} \left(- f + kC \frac{\partial^2 w_1}{\partial y^2} - \nu kC \frac{\partial^2 w_1}{\partial x^2}\right) dx\, dy$$

$$= \frac{f^2 (t_1 + t_2)}{E} \cdot 2\pi r a$$

Therefore, the partial derivative with respect to C will vanish, and

$$\frac{\partial W_3}{\partial C} = \frac{\partial W_3'}{\partial C}$$

W_3' is calculated by assuming that the average arc length of the generators after buckling equals their free length. Then

$$W_3' = f(t_1 + t_2) \int_0^{2\pi r} \int_0^{a} \left\{ \frac{C^2}{2} \left(\frac{\partial w_1}{\partial x}\right)^2 \right\} dx\, dy \qquad (25)$$

$$\frac{\partial W_3'}{\partial C} = f(t_1 + t_2)\, C \int_0^{2\pi r} \int_0^{a} \left(\frac{\partial W_1}{\partial x}\right)^2 dx\, dy$$

$$= f(t_1 + t_2)\, C \frac{\lambda^2}{r^2} \cdot \frac{a\pi r}{2} \qquad (26)$$

The Buckling Condition

The condition of instability is that equilibrium exist for C $\neq$ 0. The condition of equilibrium is that the total potential energy of the system U be a minimum. Hence,

$$\frac{\partial U}{\partial C} = 0$$

where $U = W_1 + W_2 - W_3$

$$\text{Thus} \quad \frac{\partial W_1}{\partial C} + \frac{\partial W_2}{\partial C} - \frac{\partial W_3}{\partial C} = 0 \qquad (27)$$

Now C is a factor of the left hand side of (27). Hence C = 0 is a solution and represents the unbuckled form of equilibrium. If C $\neq$ 0 the other factor must vanish and this is the condition of instability. From (1), (3), (16), (21), and (26), the buckling condition is

$$f_{cr} = \frac{E\lambda^2}{(\lambda^2 + n^2)^2} + \frac{D}{(t_1 + t_2)\, r^2} B \qquad (28)$$

where

$$B = \frac{\frac{1}{\lambda^2}(\lambda^2 + n^2)^2 \left[1 + \frac{1-\nu}{2}(n^2 + \lambda^2\theta) S_x \frac{h}{r}\right]}{1 + \left[\frac{1-\nu}{2}(n^2 + \lambda^2\theta) + \lambda^2 + n^2\theta\right] S_x \frac{h}{r} + \frac{1-\nu}{2}(\lambda^2 + n^2)^2 \theta S_x^2 \frac{h^2}{r^2}}$$

Nondimensional Buckline Coefficient

Formula (28) can be written in terms of a nondimensional buckling coefficient, K, as

$$f_{cr} = K E \frac{h}{r} \cdot \frac{2\sqrt{t_1 t_2}}{\sqrt{1-\nu^2}(t_1 + t_2)} \qquad (29)$$

and introducing the parameters

$$\xi = \frac{n^2}{\lambda^2} = \left(\frac{na}{m\pi r}\right)^2$$

$$\eta = \frac{\sqrt{1-\nu^2}(t_1 + t_2)}{\lambda^2 \frac{h}{r} 2\sqrt{t_1 t_2}} = \frac{a^2\sqrt{1-\nu^2}(t_1 + t_2)}{2m^2\pi^2 rh\sqrt{t_1 t_2}}$$

$$V_x = \frac{S_x\sqrt{1-\nu^2}(t_1 + t_2)}{2\sqrt{t_1 t_2}} = \frac{Et_c\sqrt{t_1 t_2}}{2\sqrt{1-\nu^2}\, h\, r\, G_{xz}} \qquad (30)$$

results in

$$K = \frac{\eta}{(1+\xi)^2} + \frac{(1+\xi)^2}{4\eta} \cdot \frac{1 + \frac{1-\nu}{2}(\xi + \theta)\frac{V_x}{\eta}}{1 + \frac{1-\nu}{2}(\xi + \theta)\frac{V_x}{\eta} + (1 + \xi\theta)\frac{V_x}{\eta} + \frac{1-\nu}{2}(1+\xi)^2\theta\frac{V_x^2}{\eta^2}} \qquad (31)$$

Reduction to Isotropic Core

If the core is isotropic $\theta = 1$ and the denominator of the second term of (31) can be factored, leading to

$$K = \frac{\eta}{(1+\xi)^2} + \frac{1}{4} \cdot \frac{(1+\xi)^2}{\eta + (1+\xi)V_x} \tag{32}$$

The minimum value of (32) can be found by

$$\frac{\partial K}{\partial \eta} = 0$$

from which $\eta = 1/2\,(1+\xi)^2 - (1+\xi)V_x$ (33)

and substitution into (32) results in

$$K = 1 - \frac{V_x}{1+\xi} \tag{34}$$

which is minimum, by inspection, for $\xi = 0$, hence

$$K_{min} = 1 - V_x$$

at (35)

$$\eta = \frac{1}{2} - V_x$$

Since h must be nonnegative, (35) holds for $V_x \leq 1/2$.

Limiting Cases

If G_{yz} and G_{xz} are infinite (that is, $S_x = S_y = 0$), then plane transverse cross sections of the sandwich remain plane and the sandwich construction behaves like a homogeneous shell of flexural rigidity, D, where D is given by (22).

Thus, setting $S_x = S_y = 0$ in formula (28) and making the interpretations:

$$t = t_1 + t_2$$

$$D = \frac{Et^3}{12\,(1 - \nu^2)}$$

(28) is reduced to Timoshenko's (5) solution for a homogeneous shell of thickness t (4):

$$f_{cr} = \frac{E\lambda^2}{(\lambda^2 + n^2)^2} + \frac{Et^2}{12\,(1 - \nu^2)r^2} \cdot \frac{(\lambda^2 + n^2)^2}{\lambda^2} \qquad (36)$$

It is interesting to note that in this case the buckled shape is not unique since the parameters λ and n combine into a single parameter

$$\frac{\lambda^2}{(\lambda^2 + n^2)^2}$$

which assumes a value such as to render (36) a minimum. This value corresponds to many pairs (l, n) including one for which n = 0. The conclusion is that one may as well consider only axisymmetric buckling (n = 0) since it exhibits the same critical load as the general case. See reference (5) for a complete discussion.

Another limit of interest is the case in which G_{xz} is small. When the core is soft, all sandwich structures exhibit a crimping failure associated with core shear instability. If the facings are assumed to be very thin, it has been found that this limit is simply the product of core shear stiffness and core thickness and is independent of the shape of the structure. This familiar limit is found here by letting m approach infinity, which corresponds to a large number of closely spaced crimps. Then x and h approach zero and, in the limit, using (31):

$$K = \frac{1}{4V_x} \qquad (37)$$

For the critical force per inch of edge, using the definitions of K (29) and V_x (30),

$$N_x = (t_1 + t_2) f_{cr} = \frac{h^2}{t_c} G_{xz} \approx hG_{xz} \qquad (38)$$

where it is assumed that $h \approx t_c$, i.e., that the facings are very thin.

Results and Conclusions

For $V_x = 0$ and for V_x large, the buckling coefficient is independent of $\underline{n}$, the number of circumferential waves. That is, the axisymmetric mode shape exhibits the same critical load as the general case. The question arises whether the general case exhibits a lower critical load for any value of V_x.

The answer depends upon $\underline{\theta}$. If $\theta = 1$, (34) indicates that the axisymmetric mode ($\xi = 0$) produces a minimum for all V_x. Physical reasoning shows that the same conclusion applies when $\theta < 1$, for a reduction in θ implies an increase in $\underline{G_{yz}}$ (for a given $\underline{V_x}$), and circumferential stiffening should not produce circumferential waves. A trial calculation of (31) at $V_x = .4$, $\theta = .2$ confirmed that $\xi = 0$ again produces a minimum $\underline{K}$, the same $\underline{K}$ as for $\theta = 1$.

If $\theta > 1$, the same sort of physical reasoning predicts that n should increase, and a few trial calculations with $\theta = 5$ revealed that it does. However, the reduction in critical load was surprisingly small. Even with $\underline{\theta}$ as large as 5 the buckling coefficient $\underline{K}$ was never reduced by more than 9 percent. This is because increases in $\underline{n}$ were partly offset by increases in $\underline{m}$, so that $\underline{\xi}$ remained quite small (on the order of 1). A curve for $\theta = 5$ is included in figure 3, and figure 4 shows how a typical point on this curve was arrived at by varying $\underline{\xi}$ and $\underline{\eta}$ until $\underline{K}$ was a minimum. Note that for $V_x > 1/2$ the curve for $\theta = 5$ joins the curve for $\theta \leq 1$. This is because the crimping failure produced by core shear instability depends only on $\underline{G_{xz}}$, and is therefore independent of $\underline{\theta}$.

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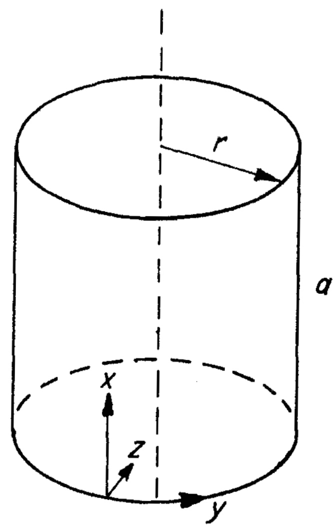

Figure 1. --Sandwich cylinder notation.

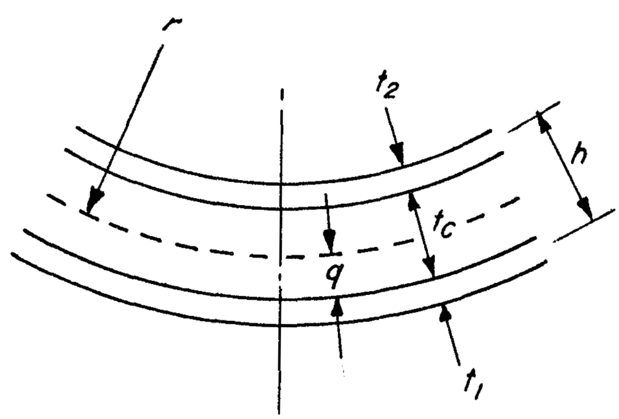

Figure 2. --Sandwich facing and core notation.

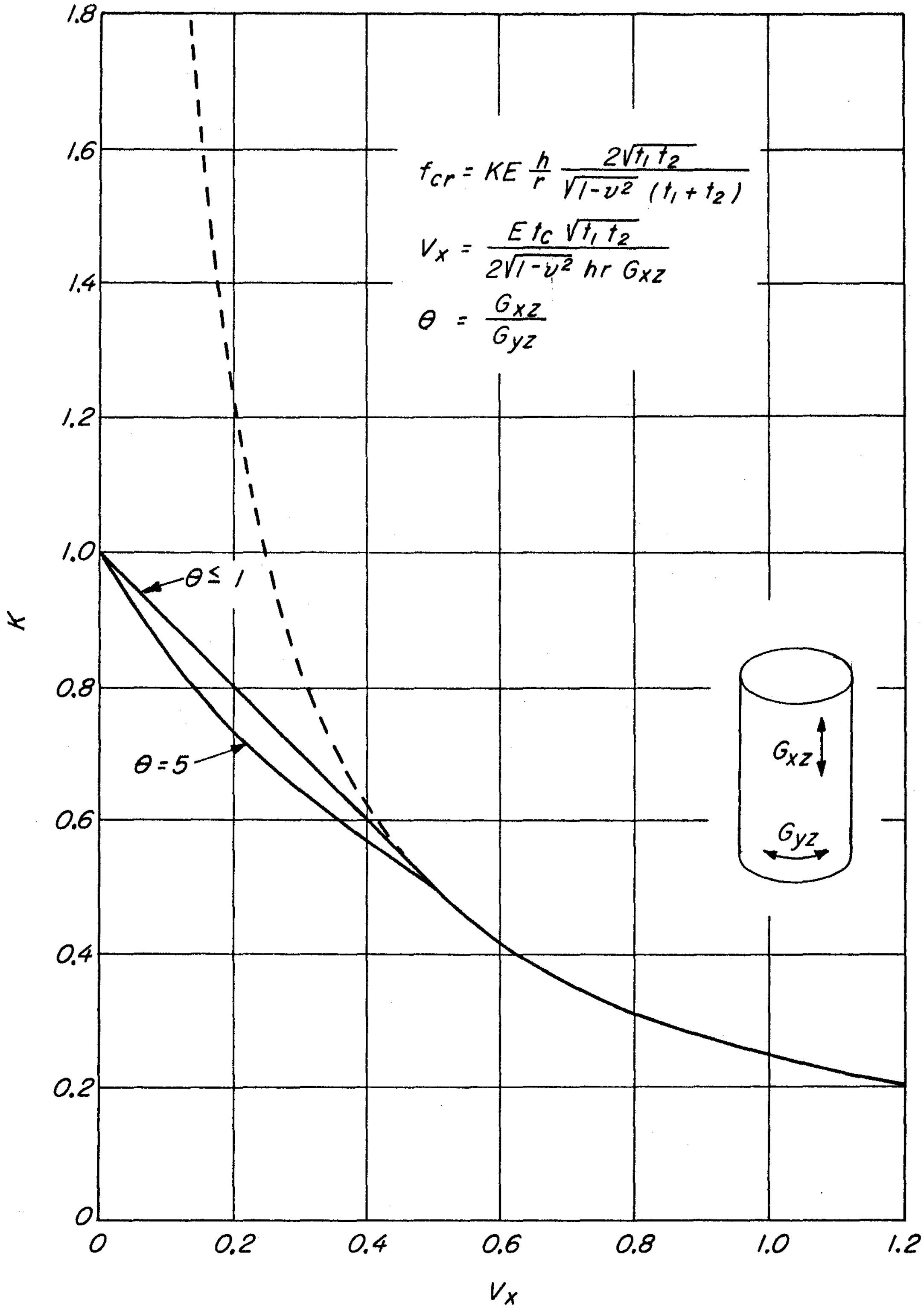

Figure 3. --Classical buckling coefficient for sandwich cylinders with isotropic facings and orthotropic core.

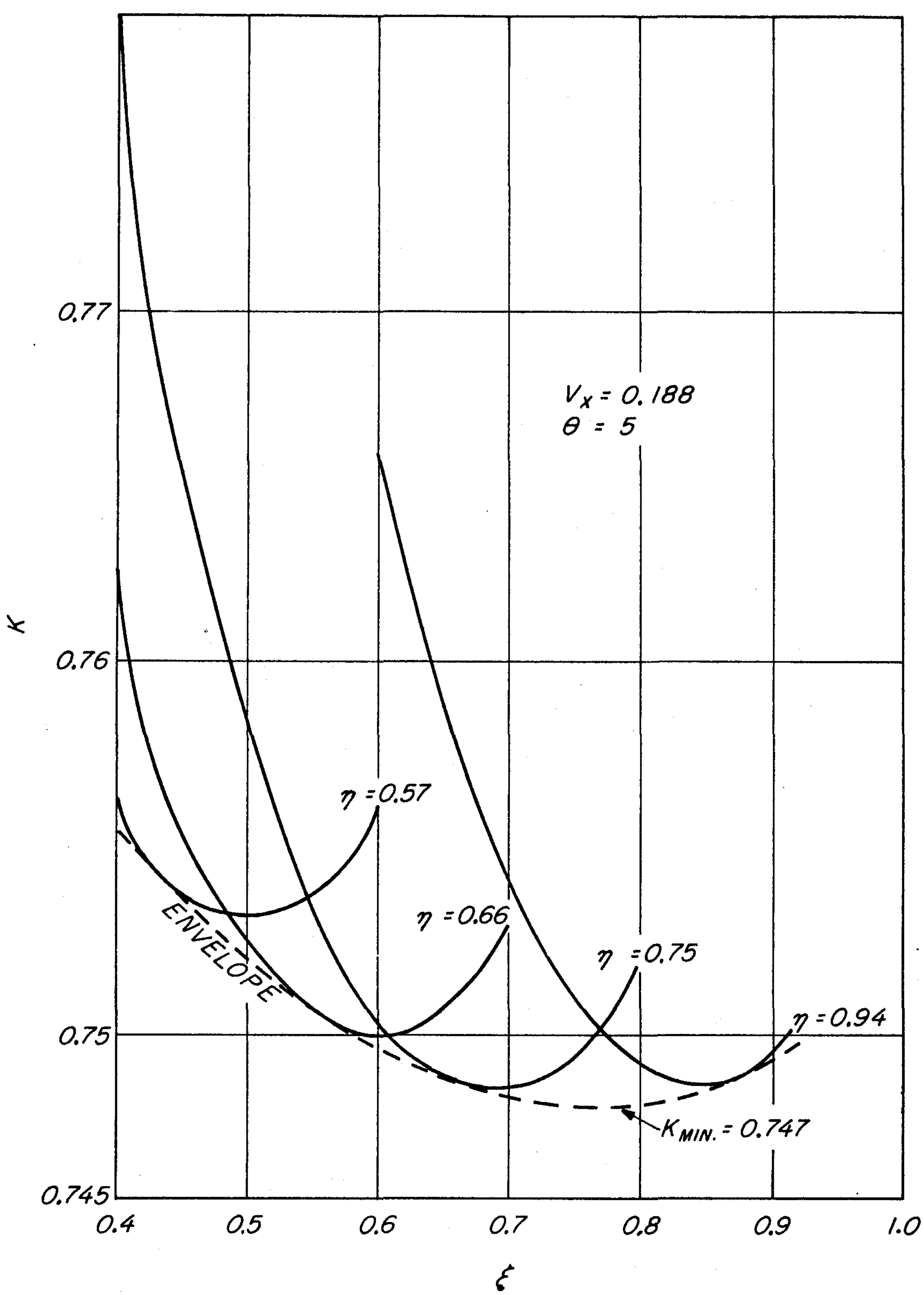

Figure 4. --Minimization of buckling coefficient.