

METHODS OF CALCULATING THE STRENGTH AND MODULUS OF ELASTICITY OF PLYWOOD IN COMPRESSION

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METHODS OF CALCULATING THE STRENGTH AND MODULUS
OF ELASTICITY OF PLYWOOD IN COMPRESSION¹

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Summary

The experimental determination of the compressive strength properties of plywood, involving hundreds of possible variations in construction and combinations of species, would require a tremendous amount of time and money. Furthermore, such a procedure would mean that each proposed plywood construction would have to be fabricated before information on its compressive strength properties could be determined. To eliminate both of these difficulties, a theoretical analysis was developed that permits computation of the compressive strengths and moduli of elasticity of plywood when the applied stress is parallel or perpendicular to the grain directions of the face plies, and the grain directions of adjacent plies are at 90° to each other. This analysis was checked by 448 compressive tests on plywood specimens. Two species of wood, in 1 veneer thickness and 4 constructions, were evaluated in this test program. Comparisons of experimental and theoretically computed results show good agreement, and indicate that the formulas developed will give accurate indications of the compressive strength properties of plywood.

Material

Test specimens were fabricated from veneer cut from Douglas-fir and Sitka spruce. Although the exact region of growth is not known, the selected

¹This report originally was issued during World War II, in cooperation with the Army-Navy-Civil Committee on Aircraft Design Criteria of the Aeronautical Board, for use by the aircraft industry. This revision evaluates the original data to substantiate the theoretical formulas.

²Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

veneer bolts were cut into rotary-cut veneer at Tacoma, Wash., in accordance with commercial practice. One 8-foot bolt was selected from each of 2 trees of each species. The veneer was of high quality and had few defects. Several thicknesses of veneer were cut from each bolt, but only one thickness, 3/16 inch, was used in this study. The veneer was clipped into consecutively numbered sheets 8 feet long and 4 feet wide, and then dried by conventional methods.

Upon arrival at the Forest Products Laboratory, the veneer was conditioned to constant weight in a room maintained at 72° F. and 52 percent relative humidity. This conditioning resulted in a moisture content of about 10 percent in the veneer. After conditioning, the veneer was cut to provide closely matched material for a set of four matched test specimens. The specimens were originally fabricated for bending tests. After the bending tests were completed, an undamaged portion of each specimen was selected for compression tests. Specimens were bonded with casein glue (Forest Products Laboratory formula 4b) with an average spread of adhesive of 0.032 pound per square foot of glue line.

The individual specimens in each matched set of four contained the same number of plies, but the ply orientation varied as follows: One specimen had the grain directions of all plies parallel to each other and parallel to the specimen length. Another specimen had similar orientation between plies, but the grain directions were perpendicular to the specimen length. In a third specimen, the grain directions of adjacent plies were at 90° to each other, with the grain direction of the outer plies parallel to the specimen length. The fourth specimen was similar in construction to the third, but the grain direction of the outer plies was perpendicular to the specimen length. Thus, two specimens might be considered to be of laminated construction, while the other two were of conventional plywood construction. The specimen orientation provided for tests on the laminated constructions both parallel and perpendicular to the grain direction and for tests on the plywood constructions both parallel and perpendicular to the grain direction of the face plies.

The number and nominal size of specimens of each species and construction tested are given in table 1. The complete program involved 448 specimens.

Method of Test

All specimens were tested with load applied parallel to the specimen length through a spherically seated loading-head. The load was applied continuously throughout the test at a rate of movement of the movable crosshead of 0.003 inch per inch of specimen length per minute. The loaded ends of the specimens were smooth, parallel to each other, and at right angles to the specimen length. Deformation data was obtained with a Lamb's roller compressometer mounted over the central portion of the

specimen length. The gage length of the compressometer was 2 inches for the 3- and 4-inch specimens and 6 inches for the 8-inch specimens. Load-deformation data were obtained throughout each test to the point of maximum load.

Theoretical Analysis

An expression for computing the effective modulus of elasticity of a plywood prism in compression developed by March³ is:

$$\bar{E}_x = \frac{1}{h} \left[\sum_{i=1}^{i=n} \frac{(E_x)_i h_i}{\lambda_i} - \frac{\sum_{i=1}^{i=n} \frac{(E_x)_i (\mu_{yx})_i h_i}{\lambda_i} \sum_{i=1}^{i=n} \frac{(E_y)_i (\mu_{xy})_i h_i}{\lambda_i}}{\sum_{i=1}^{i=n} \frac{(E_y)_i h_i}{\lambda_i}} \right] \quad (1)$$

where: \bar{E}_x = the effective modulus of elasticity in the direction parallel to the X-axis, as indicated in figure 1

$(E_x)_i$ = the modulus of elasticity of the ith ply in the X direction

$(E_y)_i$ = the modulus of elasticity of the ith ply in the Y direction

h = the overall thickness of the plywood prism

h_i = the thickness of the ith ply

$\lambda_i = 1 - (\mu_{xy})_i (\mu_{yx})_i$ which is approximately equal to unity

$(\mu_{xy})_i$ = the Poisson's ratio of the ith ply denoting the ratio of the expansion in the Y direction to the contraction in the X direction due to a compressive stress in the X direction

$(\mu_{yx})_i$ = the Poisson's ratio of the ith ply denoting the ratio of the expansion in the X direction to the contraction in the Y direction due to a compressive stress in the Y direction.

³March, H. W. Stress Strain Relations in Wood and Plywood Considered as Orthotropic Materials. Forest Products Laboratory Report No. 1503. Feb. 1944.

This expression neglects irregularities in the state of stress at the edges of the prism and assumes that the strain components are uniform across the thickness of the prism. The expression is a general one for plywood in which the grain directions of adjacent plies are parallel or perpendicular to each other and parallel or perpendicular to the edges of the prism. The plies may be of any species and of rotary-cut or quarter-sliced veneer, as long as proper consideration is given to these factors when substituting values for the elastic moduli and Poisson's ratios of the individual plies.

It can be shown that the term following the minus sign in the general expression is small and may be neglected without significant error. Substituting unity for λ_i , the expression may then be written:

$$\bar{E}_x = \frac{1}{h} \left(\sum_{i=1}^{i=n} (E_x)_i h_i \right) \quad (2)$$

This formula may be expressed in terms of the longitudinal, tangential, and radial grain directions of the individual plies. Conventional nomenclature for the effective modulus of elasticity of the plywood -- E_w when the load is applied parallel to the face grain, and E_x when the load is applied perpendicular to the face grain -- can be substituted for \bar{E}_x . The expression for the effective modulus of elasticity may then be expressed as:

$$E_w, E_x = \frac{1}{h} \sum_{i=1}^{i=n} E_i h_i \quad (3)$$

where E_i = either E_{LC} , the compressive modulus of elasticity of the individual veneers parallel to the grain; E_T , the compressive modulus of elasticity of the individual veneers in the tangential direction; or E_R , the modulus of elasticity of the individual veneers in the radial direction; depending on the orientation of the grain in the i^{th} ply and whether the veneer is rotary-cut or quarter-sliced.

If all of the veneers in the plywood are of the same species and are rotary-cut, as was the case in this study, formula (3) may be written:

$$E_w, E_x = \frac{1}{h} (E_{Lc} h_L + E_T h_T) \quad (4)$$

where h_L = total thickness of the longitudinal plies

h_T = total thickness of the tangential plies.

Formula (4) may also be expressed in terms of the ratio of the 2 moduli of elasticity.

$$E_w, E_x = \frac{E_{Lc}}{h} h_L \left(1 + \frac{E_T}{E_{Lc}} \frac{h_T}{h_L} \right) \quad (5)$$

When quarter-sliced veneer is used instead of rotary-cut veneer, the subscript T will be replaced by R and appropriate properties substituted.

The moduli of elasticity in the tangential and radial directions are not known exactly for many wood species. In these instances, an approximation may be made by assuming $\frac{E_T}{E_{Lc}}$ and $\frac{E_R}{E_{Lc}}$ to be 0.05 and 0.10 respectively.

The assumptions that the strain components are uniform across the thickness of the loaded prism and that irregularities in stress at the edges of the prism are neglected lead to the general assumption that the unit strain in the prism is equal to the unit strain in the individual plies. Since the unit strain is equal to the stress divided by the modulus of elasticity, the following expressions for stresses within the proportional limit are valid when the load is applied parallel to the face grain and perpendicular to the face grain, respectively.

$$\frac{f_{cw}}{E_w} = \frac{f_{cL}}{E_{Lc}} \quad (6)$$

and

$$\frac{f_{cx}}{E_x} = \frac{f_{cL}}{E_{Lc}} \quad (7)$$

Formulas (6) and (7) may be written:

$$f_{cw} = \frac{E_w}{E_{Lc}} f_{cL} \quad (8)$$

and

$$f_{cx} = \frac{E_x}{E_{Lc}} f_{cL} \quad (9)$$

where f_{cw} = the average stress in a plywood prism when a load is applied parallel to the face grain. It is the average stress at the proportional limit or ultimate respectively, when f_{cL} is the proportional limit strength or ultimate strength of the solid wood species in compression parallel to the grain.

f_{cx} = the average stress in a plywood prism when a load is applied perpendicular to the face grain. It is the average stress at the proportional limit or ultimate respectively, when f_{cL} is the proportional limit strength or ultimate strength of the solid wood species in compression parallel to the grain.

When formulas (8) and (9) are used to compute stresses at ultimate, it is assumed that the stress-strain curves to maximum stress for the prism and the individual parallel plies are similar, or that the ratios of the secant moduli are the same as those of the elastic moduli. Since the deformation of the prism, including that at ultimate, will be influenced primarily by the behavior of the parallel plies, this assumption appears reasonable and only small errors will be introduced through its use. When more than one species is used in the plywood, the species having the lowest ratio of $\frac{f_{cL}}{E_{Lc}}$ must be used to compute the stress in the plywood prism.

Thus formulas (5), (8), and (9) provide methods of computing the stresses and moduli of elasticity in plywood prisms. Values computed by these formulas were compared with those obtained by the tests in this program to verify the theoretical relationships.

Analysis of Data and Discussion of Results

Comparisons of theoretical and experimentally determined properties of four constructions of Sitka spruce plywood are presented in table 2, and similar data for Douglas-fir plywood are given in table 3. The first line of data under the column "Test Properties" for each construction and log lists the strength and elastic properties obtained from compression tests on specimens with all the plies parallel to each other and parallel to the direction of load application. These data are similar to those obtained from compression parallel to the grain tests on solid wood. The values for ultimate crushing strength and fiber stress at the proportional limit may be substituted for f_{cL} in formulas (8) and (9). The expression for the modulus of elasticity obtained is E_{Lc} . Test data obtained on specimens with the grain direction of all plies parallel to each other and perpendicular to the direction of applied load are given for each construction and log.

Since the veneers used in this program were rotary-cut, the expression for the modulus of elasticity is E_T . Thus the data needed to compute the strength and modulus of elasticity of plywood by means of formulas (4), (5), (8), and (9) are available. The last 2 lines in each group of 4 are the compressive properties for specimens with the grain direction of adjacent plies at right angles and the face grain direction parallel or perpendicular, respectively, to the direction of load. The average moisture content and specific gravity values shown indicate good matching between the laminated veneer and plywood specimens. Therefore, no adjustments need to be made for either of these factors when computing the plywood properties.

Using formulas (4) or (5) and (8) and (9), the average properties of each plywood construction and log were computed from the data from the laminated veneer tests. These values, as well as the ratios of computed to test values, are presented in tables 2 and 3. For a more rapid comparison of the accuracy of the theoretical formulas, the experimental and theoretical results are plotted in figure 2. Perfect agreement between theoretical and experimental results would mean that all points would lie along the 45° line in figure 2. This agreement was not obtained, but most of the test data were within 10 percent of the computed values. The envelope lines on figure 2 establish the 10 percent limits. From the data presented in figure 2 and tables 2 and 3, it is evident that the theoretical formulas give a good estimate of actual plywood properties. Thus the average strength properties of plywood of any construction can be computed from the properties of the species of wood used.

Normally, the properties of plywood in compression will be computed from the compressive properties already obtained for solid wood. Such properties may be found in U. S. Department of Agriculture Technical Bulletin No. 479, "Strength and Related Properties of Woods Grown in the United States" or the ANC-18 Bulletin, "Design of Wood Aircraft Structures." Published values for solid wood should be adjusted for the specific gravity and moisture content of the plywood according to accepted procedures before computations of plywood strength properties are made. The above publications do not give a modulus of elasticity value for compression parallel to the grain. This value can be obtained by increasing the bending modulus of elasticity by 10 percent to correct for shear deformation.

In formulas (4) and (5) the second term in the parentheses is small in comparison with the first term. Thus, approximate values can be obtained by considering only those plies with grain direction parallel to the direction of stress and omitting the second term in these formulas.

Conclusions

1. Comparisons of experimentally determined compressive strength values of plywood, with the grain direction of the face plies parallel or perpendicular to the direction of applied stress and the grain directions of adjacent plies at 90° to each other, show good agreement with compressive strength values computed with theoretically developed formulas. Therefore, these plywood properties can be determined theoretically from basic strength data already available for solid wood. This means a tremendous saving in cost and time over that which would be required if each proposed plywood construction had to be evaluated by test.

2. The effect on compressive properties of the plies with grain direction perpendicular to the direction of applied stress is small compared to the effect of the plies with grain direction parallel to the direction of applied stress. Therefore, a good approximation of the compressive properties can be obtained by considering only the plies with grain direction parallel to the applied stress direction when using the theoretical formulas.

Table 1.--Number of tests by species and construction

Species	Number of plies	Laminated construction	Plywood construction	Nominal specimen dimensions ¹
		Number of tests parallel to grain	Number of tests parallel to grain	
		to perpendicular grain	to perpendicular grain	
Sitka spruce	3	32	32	0.54 x 0.94 x 3
	5	8	8	.95 x .94 x 4
	7	8	8	1.28 x .94 x 4
	9	8	8	1.67 x 2 x 8
Douglas-fir	3	32	32	.55 x .94 x 3
	5	8	8	.92 x .94 x 4
	7	8	8	1.29 x .94 x 4
	9	8	8	1.65 x 2 x 8

¹Dimensions are tabulated as thickness, width, and length respectively.

Table 2.--Comparison of experimental and computed properties of Sikka spruce plywood tested in edgewise compression.

Number of tests	Log number of plies ²	Arrangement of plies ²	Direction of load ²	Percent moisture content	Test properties		Computed properties		Ratio		Ratio computed to test properties	
					Specific gravity	Ultimate strength: proportional limit	Ultimate strength: proportional limit	Modulus of elasticity	Modulus of elasticity	E_T/E_L	Ultimate strength: proportional limit	Modulus of elasticity
					P.S.I.	P.S.I.	P.S.I.	P.S.I.	P.S.I.	P.S.I.	P.S.I.	P.S.I.
16	1	3	0°	10.4	0.55	5,330	3,690	1,770	0.033	1,000	1,000	
16	1	3	90°	10.2	0.35	670	320	59.0				
16	1	3	90°	10.6	0.34	3,530	2,480	1,050		3,610	1,200	1.01
16	1	3	90°	10.6	0.35	1,950	1,210	555		1,900	630	1.08
16	2	3	0°	10.3	0.38	6,120	4,230	2,050	0.050			
16	2	3	90°	10.4	0.37	4,160	2,960	1,270		4,140	1,390	0.97
16	2	3	90°	10.1	0.38	2,260	1,370	625		2,160	725	0.96
4	1	5	0°	10.1	0.37	5,180	3,640	1,640	0.038			
4	1	5	90°	9.8	0.35	680	360	62.0				
4	1	5	90°	9.7	0.36	3,240	2,320	890		3,190	1,010	0.97
4	1	5	90°	9.8	0.36	2,220	1,470	620		2,190	695	1.05
4	2	5	0°	9.9	0.37	5,880	4,150	2,020	0.028			
4	2	5	90°	9.8	0.38	800	320	56.0				
4	2	5	90°	10.0	0.37	3,740	2,840	1,200		3,590	1,230	0.96
4	2	5	90°	9.8	0.38	2,640	2,010	785		2,450	840	0.93
4	1	7	0°	10.3	0.36	5,370	4,130	1,650	0.042			
4	1	7	90°	10.2	0.36	800	400	69.5				
4	1	7	90°	10.6	0.35	3,160	2,560	950		3,170	2,440	0.95
4	1	7	90°	10.3	0.36	2,500	1,810	745		2,430	745	1.03
4	2	7	0°	10.2	0.38	6,280	5,090	2,040	0.027			
4	2	7	90°	9.9	0.37	820	400	54.5				
4	2	7	90°	10.2	0.37	3,680	2,980	1,130		3,660	1,190	1.00
4	2	7	90°	10.0	0.37	2,880	2,350	910		2,790	905	0.97
3	1	9	0°	9.9	0.36	5,390	3,350	1,500	0.048			
4	1	9	90°	10.2	0.36	720	320	72.5				
4	1	9	90°	10.0	0.35	3,210	2,260	850		3,100	1,970	0.85
4	1	9	90°	10.0	0.36	2,670	1,760	755		2,540	705	0.90
4	2	9	0°	9.7	0.38	6,400	4,800	1,820	0.037			
4	2	9	90°	9.8	0.38	760	320	66.5				
4	2	9	90°	9.8	0.38	3,620	2,760	1,020		3,660	2,750	1.01
4	2	9	90°	9.9	0.38	2,910	2,080	815		2,980	845	1.07
												1.02
												1.04

¹All plies were of equal thickness.

²The grain directions of adjacent plies were either parallel (0°) for laminated construction or at 90° to each other for plywood.

³The load was applied either parallel (0°) or perpendicular (90°) to the grain direction of the face plies.

⁴Based on volume at test and oven-dry weight, exclusive of the computed weight of the glue line.

Table 3.-Comparison of experimental and computed properties of Douglas-fir plywood tested in edgewise compression.

Number of tests	Log number of plies	Ply arrangement	Direction of load ²	Test properties		Computed properties		Ratio		Ratio computed to test properties			
				Moisture content, percent	Specific gravity	Ultimate strength, P.S.I.	Fiber stress at crushing, P.S.I.	Ultimate strength, P.S.I.	Fiber stress at crushing, P.S.I.	$\frac{E_c}{E_t}$	$\frac{F_{cc}}{F_{tc}}$	Ultimate strength, P.S.I.	Fiber stress at crushing, P.S.I.
16	1	3	0°	8.7	0.49	8,660	6,470	2,490	0.029	1,000	1,000		
15	1	3	90°	8.3	.49	1,050	530	71.0		1,000	1,000		
16	1	3	90°	8.6	.48	5,570	4,150	1,570		1,000	1,680	1.06	1.07
16	1	3	90°	8.4	.48	3,220	2,280	820		1,000	880	.94	1.07
15	2	3	0°	9.8	.39	5,950	4,200	1,600	.050	1,000	1,090	.98	1.06
16	2	3	90°	9.1	.38	910	530	80.0		1,000	1,090	.98	1.06
16	2	3	90°	8.9	.38	4,130	2,710	1,030		1,000	1,090	.98	1.06
16	2	3	90°	9.2	.39	2,480	1,550	580		1,000	585	.88	1.01
4	1	5	0°	8.4	.49	8,540	5,700	2,380	.070	1,000	1,460	1.00	1.09
4	1	5	90°	8.4	.49	980	480	71.0		1,000	1,460	1.00	1.09
4	1	5	90°	8.4	.48	5,200	3,210	1,500		1,000	1,460	1.00	1.09
4	1	5	90°	8.5	.47	3,660	2,290	1,060		1,000	995	.98	1.04
4	2	5	0°	8.7	.41	6,230	4,560	1,840	.045	1,000	1,440	1.00	1.16
4	2	5	90°	8.9	.39	960	410	85.5		1,000	1,140	1.00	1.16
4	2	5	90°	8.6	.38	3,850	2,440	1,000		1,000	785	.99	1.21
4	2	5	90°	8.9	.38	2,690	1,610	615		1,000	785	.99	1.21
4	1	7	0°	8.5	.50	8,720	5,780	2,480	.028	1,000	1,450	1.03	1.06
4	1	7	90°	8.5	.47	1,060	480	68.5		1,000	1,100	.98	1.08
4	1	7	90°	8.2	.48	4,950	3,520	1,370		1,000	1,450	1.03	1.06
4	1	7	90°	8.3	.48	3,960	2,560	1,120		1,000	1,100	.98	1.08
4	2	7	0°	8.8	.39	6,060	3,940	1,660	.075	1,000	1,100	.98	1.08
4	2	7	90°	8.8	.38	970	490	91.0		1,000	990	.95	1.01
4	2	7	90°	8.6	.39	3,860	2,320	990		1,000	990	.95	1.01
4	2	7	90°	8.7	.40	3,050	1,700	725		1,000	765	.91	1.06
4	1	9	0°	7.7	.48	9,100	6,280	2,860	.029	1,000	1,440	1.00	1.04
4	1	9	90°	8.0	.48	960	330	66.5		1,000	1,280	1.05	1.09
4	1	9	90°	8.0	.48	4,920	3,780	1,170		1,000	1,280	1.05	1.09
4	1	9	90°	8.1	.48	4,170	3,070	1,000		1,000	1,040	1.00	1.04
4	2	9	0°	8.4	.37	6,060	4,050	1,440	.044	1,000	1,440	1.00	1.04
4	2	9	90°	8.5	.43	820	300	64.0		1,000	830	1.02	1.11
4	2	9	90°	8.7	.38	3,410	2,180	745		1,000	675	.95	1.01
4	2	9	90°	8.6	.40	2,990	1,890	650		1,000	675	.95	1.01

¹All plies were of equal thickness.

²The grain directions of adjacent plies were either parallel (0°) for laminated construction or at 90° to each other for plywood.

³The load was applied either parallel (0°) or perpendicular (90°) to the grain direction of the face plies.

⁴Based on volume at test and oven-dry weight, exclusive of the computed weight of the glue line.

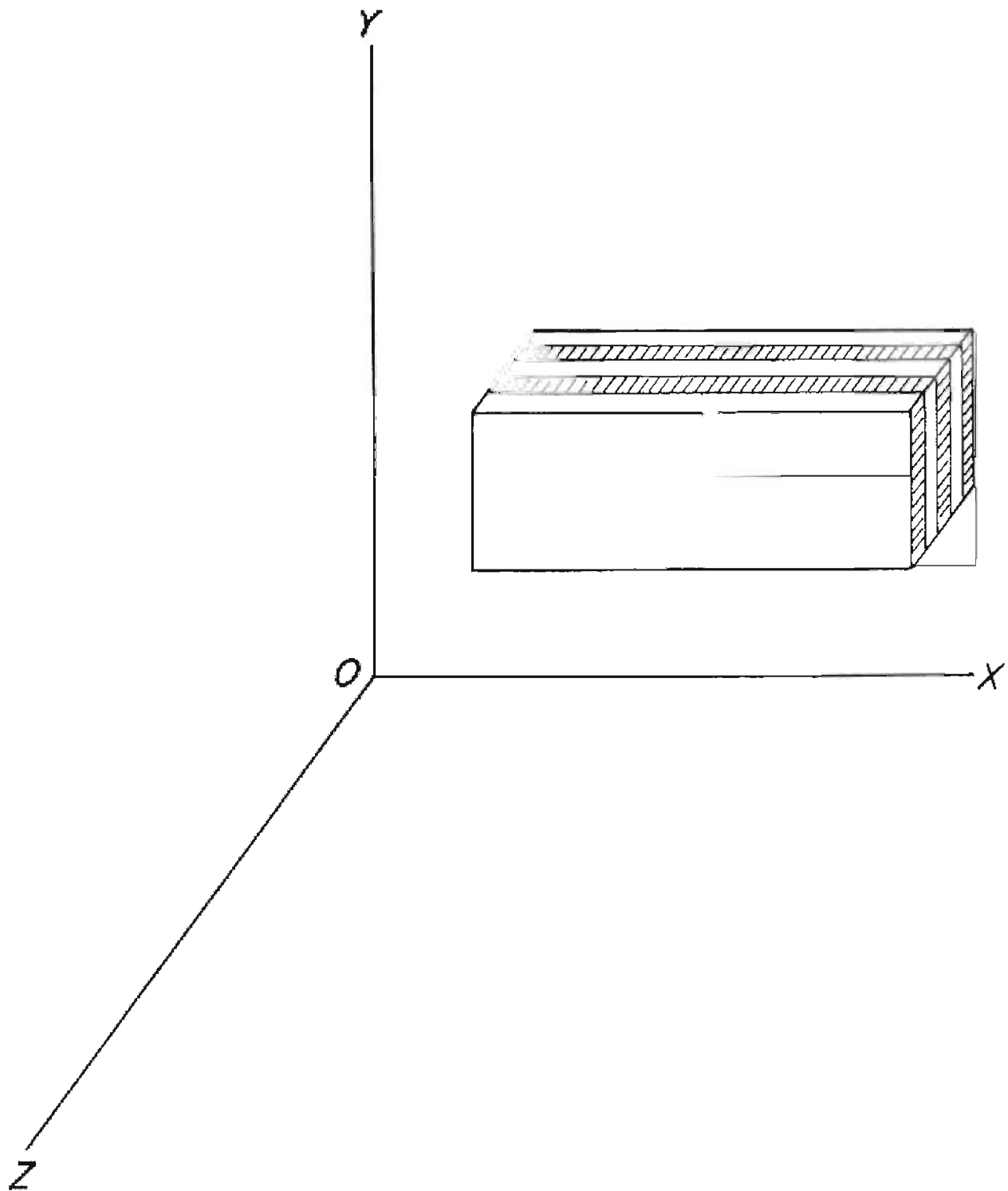


Figure 1. -- Orientation of a plywood prism with respect to its X, Y, and Z axes.

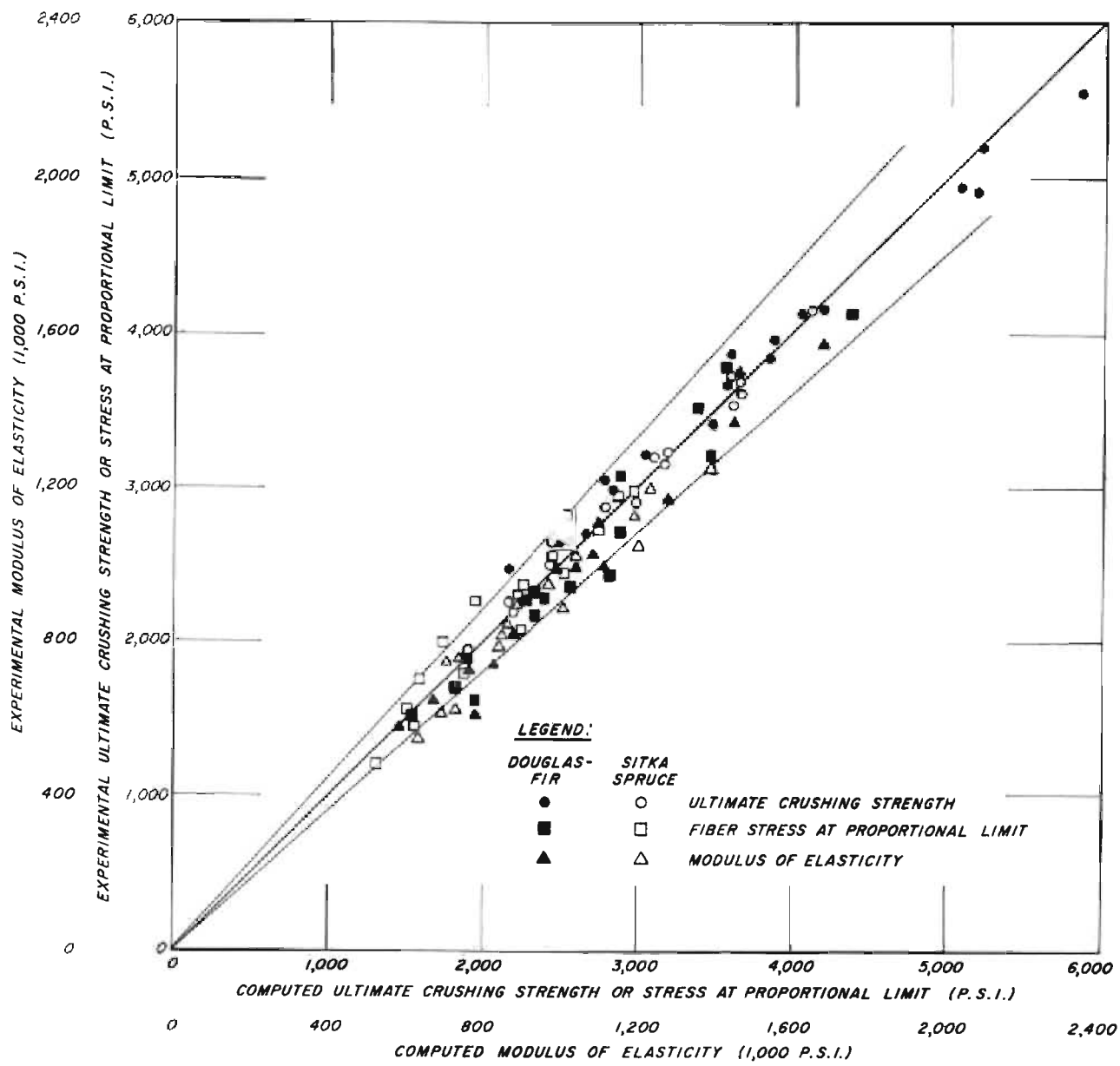


Figure 2. --Comparison of experimental and computed properties of plywood.